Localized Delaunay Triangulation with Application in Ad Hoc Wireless Networks

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Abstract—Several localized routing protocols guarantee the delivery of the packets when the underlying network topology is a planar graph. Typically, relative neighborhood graph (RNG) or Gabriel graph (GG) is used as such planar structure. However, it is well-known that the spanning ratios of these two graphs are not bounded by any constant (even for uniform randomly distributed points). Bose et al. [11] recently developed a localized routing protocol that guarantees that the distance traveled by the packets is within a constant factor of the minimum if Delaunay triangulation of all wireless nodes is used, in addition, to guarantee the delivery of the packets. However, it is expensive to construct the Delaunay triangulation in a distributed manner. Given a set of wireless nodes, we model the network as a unit-disk graph (UDG), in which a link uv exists only if the distance $\|uv\|$ is at most the maximum transmission range. In this paper, we present a novel localized networking protocol that constructs a *planar* 2.5-spanner of UDG, called the localized Delaunay triangulation (LDEL), as network topology. It contains all edges that are both in the unit-disk graph and the Delaunay triangulation of all nodes. The total communication cost of our networking protocol is $O(n \log n)$ bits, which is within a constant factor of the optimum to construct any structure in a distributed manner. Our experiments show that the delivery rates of some of the existing localized routing protocols are increased when localized Delaunay triangulation is used instead of several previously proposed topologies. Our simulations also show that the traveled distance of the packets is significantly less when the FACE routing algorithm is applied on LDEL, rather than applied on GG.

Index Terms—Delaunay triangulation, localized method, planar, routing, spanner, topology control, wireless networks.

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1 INTRODUCTION

W^E consider a wireless ad hoc network (or sensor network) with all nodes distributed in a two-dimensional plane. Assume that all wireless nodes have distinctive identities and each static wireless node knows its position information,[1] either through a low-power Global Position System (GPS) receiver or through some other way. For simplicity, we also assume that all wireless nodes have the same maximum transmission range and we normalize it to one unit. By one-hop broadcasting, each node u can gather the location information of all nodes within the transmission range of u. Consequently, all wireless nodes V together, define a unit-disk graph $UDG(V)$, which has an edge uv if and only if the Euclidean distance $\|uv\|$ between u and v is less than one unit. Throughout this paper, a *broadcast* by a node u means u sends the message to all nodes within its transmission range. In wireless ad hoc networks, the radio signal sent out by a node u can be received by all nodes within the transmission range of u. The main communication cost in wireless networks is to send out the signal while the receiving cost of a message is

neglected here. Consequently, throughout this paper, we are interested in designing a protocol with a small total number of messages sent out by all wireless nodes.

One of the key challenges in the design of ad hoc networks is the development of dynamic routing protocols that can efficiently find routes between two communication nodes. In recent years, a variety of routing protocols [1], [2], [3], [4], [5], [6] targeted specifically for ad hoc environment have been developed. See [7], [8] for a review of most routing protocols.

Several researchers proposed another set of routing protocols, namely, the localized routing, which selects the next node to forward the packets based on the information in the packet header and the position of its local neighbors. Bose and Morin [9] showed that several localized routing protocols guarantee to deliver the packets if the underlying network topology is the Delaunay triangulation of all wireless nodes. They also gave a localized routing protocol based on the Delaunay triangulation such that the total distance traveled by the packet is no more than a small constant factor of the distance between the source and the destination. However, it is expensive to construct the Delaunay triangulation in a distributed manner, and routing based on it might not be possible since the Delaunay triangulation can contain links longer than one unit.

Several researchers also proposed to use some *planar* network topologies that can be constructed efficiently in a distributed manner. Lin et al. [10] proposed the first localized algorithm that guarantees delivery by memorizing past traffic at nodes. Bose et al. [11] proposed to use the Gabriel graph as the underlying structure for the FACE routing method. Subsequently, Karp et al. [12] discussed, in detail, the medium access layer and conducted experiments

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with moving nodes for the Face routing method. Barriére et al. [13] extended the scheme on graphs which are fuzzy unit graphs; that is, two nodes are connected if their distance is at most r, not connected if the distance is at least R, and may be connected otherwise. They showed that their algorithm works correctly if $R \leq \sqrt{2}r$. Routing, according to the right-hand rule, which guarantees delivery in planar graphs [9], is also used when simple greedy-based routing heuristics fail.

However, it is well-known [14], [15] that the spanning ratios of both RNG and GG are not bounded by any constant. Here, given a graph H, a spanning subgraph G of H is a t-spanner if the length of the shortest path connecting any two points in G is no more than t times the length of the shortest path connecting the two points in H. Moreover, it was shown by Bose et al. [14] that the spanning ratio of the Gabriel graph on a uniformly random n points set in a square is almost surely at least $\Omega(\sqrt{\log n / \log \log n})$. Thus, no matter how good the routing method is, the spanning ratio achieved by applying the method on the Gabriel graph or on the relative neighborhood graph is at least $\Omega(\sqrt{\log n / \log \log n})$, almost surely. Consequently, to make a localized routing protocol efficient, we need to construct a *planar spanner* locally. Here, a structure G can be constructed locally if every wireless node u can decide the edges of G incident on u, by using only the information of nodes within a constant hops. In this paper, we design a localized algorithm that constructs a planar t-spanner for the unit-disk graph, such that some of the localized routing protocols can be applied on it. We obtain a value of approximately 2.5 for the constant t. Notice that the spanning ratio achieved by a specific routing method could be much larger than the spanning ratio of the underlying structure. Nonetheless, a structure with a small spanning ratio is necessary for some routing method to possibly perform well.

Our structure is based on the Delaunay triangulation. Given a set of points V, the *unit Delaunay triangulation*, denoted by $UDel(V)$, is the graph obtained by removing all edges of the Delaunay triangulation $Del(V)$ that are longer than one unit. It was proven in [16], [17] that $UDel(V)$ is a t-spanner of $UDG(V)$. We then give a localized algorithm that constructs a graph, called localized Delaunay graph $LDel^{(1)}(V)$. We prove that $LDel^{(1)}(V)$ is a t-spanner by showing that it is a supergraph of $UDel(V)$. Additionally, we prove that $LDel^{(1)}(V)$ has thickness two, i.e., it can be decomposed to two planar graphs. We then show how to make the graph $LDel^{(1)}(V)$ planar efficiently without losing the spanner property. The total communication cost of our approach is $O(n \log n)$ bits, which is optimal within a constant factor. Notice that every node has to send at least one message to its neighbors to notify its existence in any protocol, which implies that the communication cost is at least $n \log n$ bits for any protocol. We assume a node ID can be represented by $\log n$ bits.

The precise worst-case upper bound on the communication costs to construct our planar spanner is $(37q + 13p + 100)n$ bits, where q is the number of bits required to represent the unique node IDs, and p is the number of bits required to represent the geometric position of a node. Our method operates in an asynchronous environment, and we

do not count the time it takes to build the structure. We do not have nontrivial worst-case upper bounds on maintaining the structure due to changes in the network configuration, but believe the average case (assuming changes are random) is acceptable due to the fact that, because the structure is planar and planar graphs have at most $3n - 6$ edges, the average degree of a node in the structure is at most 6. Our belief is also based on the fact that inserting nodes in a Delaunay triangulation in a random order results in a $O(n \log n)$ expected-time centralized algorithm [23]. When a node moves, the expected number of edges that are affected is at most 6. Thus, the average cost of updating the structure is small.

The Gabriel graph can be constructed with $(q + p + 1)n$ bits and is very easy to maintain when the network changes. Thus, there is a trade off between the better worst-case spanning ratio when applying the greedy routing schemes on our structure and the higher cost of constructing and maintaining it, compared to the Gabriel graph.

Previously, there were some approaches proposed to approximate the Delaunay triangulation locally. Hu [18] used the Delaunay triangulation to configure the wireless network topology such that a planar graph with a bounded node degree is computed. A major step in his method is that each node u computes *all* Delaunay edges whose length is no more than the transmission range. It used the Voronoi diagram of node u to compute all such Delaunay edges. However, this approach will not always work. A simple observation is that, in order to determine whether an edge uv belongs to the Delaunay triangulation, we have to check whether certain circles passing through u and v are empty (not containing wireless nodes in their interior). Obviously, in the worst case, the circumradius of such a circle could be infinity even when the edge length uv is bounded, implying that we may have to check all nodes. Due to space limit, we omit the detail of why the method in [18] will not work. Moreover, it is unknown whether Hu's structure has a constant spanning ratio.

A related result was published by Gao et al. [17]. The conference version of this paper, [16], was obtained independently and was submitted before [17] was published. Gao et al. [17] proposed another structure, called *restricted Delaunay graph* RDG and showed that it has good spanning ratio properties and described how to maintain it locally. They called any planar graph containing $UDel(V)$ a restricted Delaunay graph. They described a distributed algorithm to maintain a RDG such that, at the end of the algorithm, each node u maintains a set of edges $E(u)$ incident to u. Those edges $E(u)$ satisfy that 1) each edge in $E(u)$ has length at most one unit; 2) the edges are consistent, i.e., an edge $uv \in E(u)$ if and only if $uv \in E(v)$; 3) the graph obtained is planar; and 4) $UDel(V)$ is in the union of all edges $E(u)$.

Their algorithm works as follows: First, each node u acquires the position of its 1-hop neighbors $N_1(u)$ and computes the Delaunay triangulation $Del(N_1(u))$ on $N_1(u)$, including u itself. In the second step, each node u sends $Del(N_1(u))$ to all of its neighbors. Let $E(u) = \{uv \mid uv \in Del(N_1(u))\}$. For each edge $uv \in E(u)$ and for each $w \in N_1(u)$, if u and v are in $N_1(w)$ and $uv \notin Del(N_1(w))$, then node u deletes edge uv from $E(u)$. They proved that, when the above steps are finished, the resulting edges $E(u)$ satisfy the four

properties listed above. The communication cost could be as large as $\Theta(n^2)$, and the computation cost could be as large as $\Theta(n^3)$, which are much higher than ours.

Recently, Li et al. [19] also proposed another structure *partial Delaunay triangulation* (PDT), which is a subset of $UDel(V)$ and a superset of Gabriel graph, for scatternet formation in Bluetooth network. Unfortunately, PDT does not have constant bounded spanning ratio.

Bose et al. [11] and Karp et al. [12] proposed similar algorithms that route the packets using the Gabriel graph to guarantee the delivery. Applying the routing methods proposed in [11], [12] on the planarized localized Delaunay graph $LDel^{(1)}(V)$, a better performance is expected because the localized Delaunay triangulation is denser compared to the Gabriel graph, but still with $O(n)$ edges. Our simulations show that the delivery rates of several localized routing protocols are increased when the localized Delaunay triangulation is used. In our experiments, several simple local routing heuristics, applied on the localized Delaunay triangulation, have always successfully delivered the packets, while other heuristics were successful in more than 90 percent of the random instances. The greedy-based localized routing schemes may still fail to deliver the packet on localized Delaunay triangulation. Because the constructed topology is planar, we can use the right-hand rule or Face routing to guarantee the delivery of the packets from source node to the destination when simple heuristics fail. The experiments also show that several localized routing algorithms (notably, compass routing [20] and greedy routing) also result in a path whose length is within a small constant factor of the shortest path; we already know such a path exists since the localized Delaunay triangulation is a t-spanner. Notice that there is a difference between the spanning ratio of the underlying structure and the spanning ratio achieved by a specific routing algorithm. Obviously, *any* routing method cannot achieve small spanning ratio when it is applied on a structure with large spanning ratio. We also conduct extensive simulations of the Face routing method [11] and the Greedy Face Routing method (applies the greedy routing whenever possible and uses the Face routing if local minimum occurs) on the localized Delaunay triangulation and the Gabriel graph. For Face routing, we found that the worst-case spanning ratio when localized Delaunay triangulation is used is significantly less than the spanning ratio when Gabriel graph is used, although the average spanning ratio achieved by these two structures are almost the same when the network has no more than 100 nodes. We expect our local Delaunay triangulation to perform better when the number of nodes is significantly large, since the spanning ratio of the Gabriel graph on a set of n random points is $\Omega(\sqrt{\log n / \log \log n})$.

The remainder of the paper is organized as follows: In Section 2, we review some structures that are often used to construct the topology for wireless networks. We define localized Delaunay triangulations $LDel^{(k)}(V)$ and study their properties in Section 3. Section 4 presents the first localized efficient algorithm that constructs a planar graph, $PLDel(V)$, which contains $UDel(V)$ as a subgraph. Thus, $PLDel(V)$ is a planar t-spanner. The correctness of our algorithm is justified in the Appendix. We demonstrate the effectiveness of the localized Delaunay triangulation in Section 5, by studying the performance of various routing protocols on it. We conclude our paper and discuss possible future research directions in Section 6.

2 PRELIMINARIES

2.1 Spanner

Let $\Pi_G(u,v)$ be the shortest path connecting u and v in a weighted graph G and $\|\Pi_G(u,v)\|$ be the length of $\Pi_G(u,v)$. Then, a graph G is a t-spanner of a graph H if $V(G) = V(H)$ and, for any two nodes u and v of $V(H)$, $\|\Pi_H(u,v)\| \leq \|\Pi_G(u,v)\| \leq t\|\Pi_H(u,v)\|$. With H understood, we also call t the *length stretch factor* of the spanner G.

There are several geometrical structures which are proven to be t-spanners for the Euclidean complete graph $K(V)$ of a point set V. For example, the Yao graph [21] and the θ-graph [22] have been shown to be t-spanners. However, both of these two geometrical structures are not guaranteed to be planar in two dimensions.

Let $\varrho_G(u,v)$ be the path found by a unicasting routing method ϱ from node u to v in a weighted graph G and $\|\varrho_G(u,v)\|$ be the length of the path. The spanning ratio achieved by a routing method ϱ is defined as $\max_G \max_{u,v} \|\varrho_G(u,v)\| / \|uv\|$.

2.2 Delaunay Triangulation

We review the definition of Delaunay triangulation [23]. We assume that all wireless nodes are given as a set V of n nodes in a two-dimensional space. Each node has some computational power. We also assume that there are not four nodes of V that are cocircular. A triangulation of V is a *Delaunay triangulation*, denoted by $Del(V)$, if the circumcircle of each of its triangles does not contain any other nodes of V in its interior. A triangle is called the *Delaunay triangle* if its circumcircle is empty of nodes of V inside. It is well-known that the Delaunay triangulation $Del(V)$ is a planar t-spanner of the completed Euclidean graph $K(V)$. This was first proven by Dobkin et al. [24] with upper bound $\frac{1+\sqrt{5}}{2}\pi \approx 5.08$ on t. Then, Kevin and Gutwin [25], [22] improved the upper bound on t to be $\frac{2\pi}{3\cos\frac{\pi}{6}} = \frac{4\sqrt{3}}{9}\pi \approx 2.42$. The best-known lower bound on t is $\pi/2$, which is due to Chew [26], and it is widely believed to be the actual upper bound also.

2.3 Proximity Graphs

Besides the Delaunay triangulation, various proximity subgraphs of UDG can be defined [27], [28], [29], [30], [21] over a set of n two-dimensional wireless nodes V.

For convenience, let $disk(u,v)$ be the closed disk with diameter uv, let $disk(u,v,w)$ be the circumcircle defined by the triangle $\triangle uvw$, and let $B(u,r)$ be the circle centered at u with radius r. Let $x(v)$ and $y(v)$ be the value of the x-coordinate and y-coordinate of a node v, respectively.

The *constrained relative neighborhood graph*, denoted by $RNG(V)$, consists of all edges uv such that $\|uv\| \leq 1$ and there is no point $w \in S$ such that $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. The *constrained Gabriel graph*, denoted by $GG(V)$, consists of all edges uv such that $\|uv\| \leq 1$ and the interior of $disk(u,v)$ does not contain any node from V. The *constrained Yao graph* with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(V)$, is

defined as follows: At each node u, any k equal-separated rays originated at u define k cones. In each cone, choose the closest node v to u with distance at most one, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily. Let $YG_k(V)$ be the undirected graph obtained by ignoring the direction of each link in $\overrightarrow{YG}_k(V)$.

The length stretch factor of $RNG(V)$ is at most $n-1$, see [14]; and the length stretch factor of $GG(V)$ is at most $\sqrt{n-1}$, see [31]. Several papers [32], [33], [27] have shown that the Yao graph $YG_k(V)$ has length stretch factor at most $\frac{1}{1-2\sin\frac{\pi}{k}}$. However, the Yao graph is not guaranteed to be planar. The relative neighborhood graph and the Gabriel graph are planar graphs, but they are not spanners for the unit-disk graph. In this paper, we are interested in locally constructing a planar graph that is a spanner of the unit-disk graph.

2.4 Localized Routing Algorithms

Let $N_k(u)$ be the set of nodes of V that are within k hops of u in the unit-disk graph $UDG(V)$. A node $v \in N_k(u)$ is called the k-neighbor of the node u. Usually, here, the constant k is 1 or 2, which will be omitted if it is clear from the context. In this paper, we always assume that each node u of V knows its location and identity. Then, after one *broadcast* by every node, each node u of V knows the location and identity information of all nodes in $N_1(u)$. The total communication cost of all nodes to do so is $O(n \log n)$ bits.

A distributed algorithm is a *localized algorithm* if it uses only the information of all k-local nodes of each node plus the information of a constant number of additional nodes. In this paper, we concentrate on the case $k = 1$. That is, a node uses only the information of the 1-hop neighbors. A graph G can be constructed locally in the ad hoc wireless environment if each wireless node u can compute the edges of G incident on u by using only the location information of all its k-local nodes. In this paper, we design a localized algorithm that constructs a planar t-spanner for the unit-disk graph $UDG(V)$ such that some localized routing protocols can be applied on it. The localized construction of the structure is attractive for wireless ad hoc networks due to efficient updating of the structure in mobile environment.

Assume a packet is currently at node u, and the destination node is t. Several localized routing algorithms that use just the local information of u to route packets (i.e., find the next node v of u) were developed. Kranakis et al. [20] proposed to use the compass routing, which basically finds the next relay node v such that the angle $\angle vut$ is the smallest among all neighbors of u in a given topology. Lin et al. [10], Bose et al. [11], and Karp et al. [12] proposed similar greedy routing methods, in which node u forwards the packet to its neighbor v in a given topology which is closest to t. Recently, Bose et al. [34], [9], [11] proposed several localized routing algorithms that route a packet from a source node s to a destination node t. Specifically, Bose and Morin [9] proposed a localized routing method based on the Delaunay triangulation. They showed that the distance traveled by the packet is within a small constant factor of the distance between s and t. They also proved that the compass routing and the greedy routing

method guarantee to deliver the packet if the Delaunay triangulation is used.

3 LOCAL DELAUNAY TRIANGULATION

In this section, we define a new topology, called local Delaunay triangulation, which can be constructed in a localized manner. We first introduce some geometric structures and notations to be used in this section. All angles are measured in radians and take values in the range $[0, \pi]$. For any three points p_1, p_2, and p_3, the angle between the two rays p_1p_2 and p_1p_3 is denoted by $\angle p_3p_1p_2$ or $\angle p_2p_1p_3$. The *closed* infinite area inside the angle $\angle p_3p_1p_2$, also referred to as a sector, is denoted by $\angle p_3p_1p_2$ without confusion to the angle notation. The triangle determined by p_1, p_2, and p_3 is denoted by $\triangle p_1p_2p_3$.

An edge uv is called constrained *Gabriel edge* (or simply, Gabriel edge here) if $\|uv\| \leq 1$ and the open disk using uv as diameter does not contain any node from V. It is well-known [23] that the constrained Gabriel graph is a subgraph of the Delaunay triangulation, more precisely, $GG(V) \subseteq UDel(V)$. Recall that a triangle $\triangle uvw$ belongs to the Delaunay triangulation $Del(V)$ if its circumcircle $disk(u, v, w)$ does not contain any other node of V in its interior. To simplify the proofs, from now on, we assume that there are no four nodes of V cocircumcircle. If four nodes are on the same circle, a very small random perturbation to their coordinates allows the assumption above without causing any problems in the actual network. It is easy to show that nodes u, v, and w together cannot decide if they can form a triangle $\triangle uvw$ in $Del(V)$ by using only their local information. We say a node x can see another node y if $\|xy\| \leq 1$. The following definition is one of the key ingredients of our localized algorithm.

Definition 1. *A triangle $\triangle uvw$ satisfies k-localized Delaunay property if the interior of the circumcircle $disk(u, v, w)$ does not contain any node of V that is a k-neighbor of u, v, or w; and all edges of the triangle $\triangle uvw$ have a length of no more than one unit. Triangle $\triangle uvw$ is called a k-localized Delaunay triangle.*

Definition 2. *The k-localized Delaunay graph over a node set V, denoted by $LDel^{(k)}(V)$, has exactly all Gabriel edges and the edges of all k-localized Delaunay triangles.*

When it is clear from the context, we will omit the integer k in our notation of $LDel^{(k)}(V)$. Our original conjecture was that $LDel^{(1)}(V)$ is a planar graph and, thus, we can easily construct a planar t-spanner of $UDG(V)$ by using a localized approach. Unfortunately, as we will show later, the edges of the graph $LDel^{(1)}(V)$ may intersect. While $LDel^{(1)}(V)$ is a t-spanner, its construction is a little bit more complicated than some other nonplanar t-spanners, such as the Yao structure [21] and the θ-graph [22]. But, we can make $LDel^{(1)}(V)$ planar efficiently, a result we describe later in this paper.

The k-localized *Delaunay graph* $LDel^{(k)}(V)$ over a node set V satisfies a monotone property: $LDel^{(k+1)}(V)$ is always a subgraph of $LDel^{(k)}(V)$ for any positive integer k.

3.1 $LDel^{(k)}(V)$ is a t-Spanner
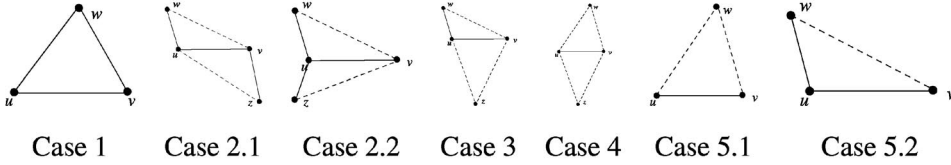
The following lemma was proven by us in [16].

Fig. 1. The neighborhood configuration of edge uv. Dashed lines (solid lines) denote edges with length $> 1 (\le 1)$.

Lemma 1. *Graph $UDel(V)$ is a t-spanner of $UDG(V)$ for $t \le 2.42$.*

Theorem 1. *Graph $UDel(V)$ is a subgraph of the k-localized Delaunay graph $LDel^{(k)}(V)$.*

Proof. We prove the theorem by showing that each edge uv of the unit Delaunay triangulation graph $UDel(V)$ appears in the localized Delaunay graph $LDel^{(k)}(V)$. For each edge uv of $UDel(V)$, the following five cases are possible (see Fig. 1 for illustrations).

Case 1: There is a triangle $\triangle uvw$ incident on uv such that all edges of $\triangle uvw$ have length at most one unit. Because the circumcircle $disk(u, v, w)$ is empty of nodes of V, triangle $\triangle uvw$ satisfies the k-localized Delaunay property and, thus, edge uv belongs to $LDel^{(k)}(V)$.

Case 2: Each of the two triangles incident on uv has only one edge longer than one unit.

Case 3: One triangle $\triangle uvw$ incident on uv has only one edge with length larger than one unit and the other triangle $\triangle uvz$ has two edges with length larger than one unit.

Case 4: Each of the two triangles incident on uv has two edges longer than one unit.

We prove Cases 2, 3, and 4 together. Assume the two triangles are $\triangle uvw$ and $\triangle uvz$. Let $H_{uv,w}$ be the half-plane that is divided by uv and contains node w. Then, edge uv is not the longest edge in triangle $\triangle uvw$ and, thus, the angle $\angle uwv < \frac{\pi}{2}$. This implies that the circumcircle $disk(u, v, w)$ contains $disk(u, v) \cap H_{uv,w}$. Similarly, the other half of $disk(u, v)$ is contained inside the circumcircle $disk(u, v, z)$. Both $disk(u, v, w)$ and $disk(u, v, z)$ do not contain any node of V inside. It implies that $disk(u, v)$ is empty, i.e., edge uv is a Gabriel edge. Consequently, edge uv will be inserted to $LDel^{(k)}(V)$.

Case 5: There is only one triangle incident on uv and it has at least one edge with length larger than one unit. Similar to Cases 2, 3, and 4, we can show that $disk(u, v)$ is empty and, therefore, edge uv will be inserted to $LDel^{(k)}(V)$ as a Gabriel edge. $\square$

3.2 $LDel^{(1)}(V)$ May Be Nonplanar

The definition of the 1-localized Delaunay triangle does not prevent two triangles from intersecting or prevent a Gabriel edge from intersecting a triangle. Fig. 2 gives such an example with six nodes $\{u, v, w, x, y, z\}$ that $LDel^{(1)}(V)$ is not a planar graph. Here, $\|uv\| = \|xy\| = 1$, and $\|uy\| = \|vy\| > 1$. Node x is out of circumcircle of $disk(u, v, w)$. Triangle $\triangle uvw$ is a 1-localized Delaunay triangle. If the node z does not exist, edge xy is a Gabriel edge. The triangle $\triangle uvw$ intersects the Gabriel edge xy if z does not exist, otherwise, it intersects the 1-localized Delaunay triangle $\triangle xyz$. The example illustrated by Fig. 2

also implies that a triangle in $LDel^{(1)}(V)$ can intersect many other edges (by creating equal-length Gabriel edges $x_1y_1, x_2y_2, \cdots$, which are parallel to Gabriel edge xy).

3.3 $LDel^{(1)}(V)$ Has Thickness 2

In this section, we claim that $LDel^{(1)}(V)$ has thickness two, or, in other words, its edges can be partitioned in two planar graphs. From Euler's formula, it follows that a simple planar graph with n nodes has at most $3n - 6$ edges and, therefore, $LDel^{(1)}(V)$ has at most $6n$ edges. The proof of the following theorem is in the Appendix.

Theorem 2. *Graph $LDel^{(1)}(V)$ has thickness 2.*

Our construction algorithm of $LDel^{(1)}(V)$ below also implies that $LDel^{(1)}(V)$ has a linear number of edges. Theorem 2 gives a better constant.

3.4 $LDel^{(k)}(V)$, $k \ge 2$, Is Planar

Although we gave an example showing that $LDel^{(1)}(V)$ is not always a planar graph, we will show that $LDel^{(k)}(V)$, $k \ge 2$, is always planar. Theorem 1 implies that each edge uv of $UDel(V)$ is either a Gabriel edge or forms a 1-localized Delaunay triangle with some edges from $UDel(V)$. Obviously, any two edges in $UDel(V)$ do not intersect. Thus, each possible intersection in $LDel^{(k)}(V)$ is caused by at least one edge of some localized Delaunay triangle. We begin the proof that $LDel^{(k)}(V)$, $k \ge 2$, is planar by giving some simple facts and lemmas.

Remark. *If a Gabriel edge uv intersects an edge xy, then xy does not belong to $UDel(V)$.*

Lemma 2. *If a Gabriel edge uw intersects a localized Delaunay triangle $\triangle xyz$, then u and w cannot be both outside the circumcircle $disk(x, y, z)$.*

Proof. Let c be the circumcenter of the triangle $\triangle xyz$. Then, at least one of the x, y, and z must be on the different side of line uw versus the center c; say x. If both u and w are outside the circumcircle $disk(x, y, z)$, then $\angle wxu > \frac{\pi}{2}$.
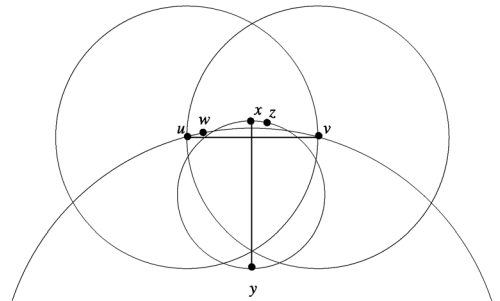

Fig. 2. $LDel^{(1)}(V)$ is not planar.

Thus, x is inside $disk(u, w)$, which contradicts that uw is a Gabriel edge. □

Lemma 3. *Let $\triangle uvw$ and $\triangle xyz$ be two triangles of $LDel^{(k)}(V)$, $k \geq 1$, and assume the edge uw intersects the triangle $\triangle xyz$ and that $disk(u, v, w)$ does not contain any of the nodes of $\{x, y, z\}$. Then, $disk(x, y, z)$ contains either u or w.*

See the Appendix for the proof. The above lemma guarantees that, if two k-localized Delaunay triangles $\triangle uvw$ and $\triangle xyz$ intersect, then either $disk(u, v, w)$ or $disk(x, y, z)$ violates the Delaunay property by just considering the nodes $\{u, v, w, x, y, z\}$.

The following theorem gives more insight regarding localized Delaunay graphs and it is never used later in the paper. Its proof is immediate from Theorem 5.

Theorem 3. *$LDel^{(2)}(V)$ is a planar graph.*

In conclusion, we defined a sequence of localized Delaunay graphs $LDel^{(k)}(V)$, where $1 \leq k \leq n$. All graphs are t-spaner of the unit-disk graph with the following properties:

- $UDel(V) \subseteq LDel^{(k)}(V)$, for all $1 \leq k \leq n$.
- $LDel^{(k+1)}(V) \subseteq LDel^{(k)}(V)$, for all $1 \leq k \leq n$.
- $LDel^{(k)}(V)$ are planar graphs for all $2 \leq k \leq n$.
- $LDel^{(1)}(V)$ is not always planar.

4   LOCALIZED ALGORITHM

In this section, we study how to locally construct a planar t-spanner of $UDG(V)$. We use q to denote the number of bits required to represent the unique node IDs and p to denote the number of bits required to represent the geometric position of a node. When using big-Oh notation, we make the reasonable assumption that both q and p are $O(\log n)$.

Although the graph $UDel(V)$ is a t-spanner for $UDG(V)$, it cannot be constructed locally. We can construct $LDel^{(2)}(V)$, which is guaranteed to be a planar spanner of $UDel(V)$, but the total communication cost of this approach could be $O(m \log n)$ bits, where m is the number of edges in $UDG(V)$ and could be as large as $O(n^2)$. In order to reduce the total communication cost to $O(n \log n)$ bits, we do not construct $LDel^{(2)}(V)$ and, instead, we extract a planar graph $PLDel(V)$ out of $LDel^{(1)}(V)$.

4.1   Algorithm

Recall that $LDel^{(1)}(V)$ is not guaranteed to be a planar graph. Our algorithm first constructs $LDel^{(1)}(V)$ and then removes edges from it to make it planar. The result of the algorithm is called $PLDel(V)$ and we show later that it is planar and that it contains $UDel(V)$ as a subgraph (and, therefore, it is a t-spanner of $UDG(V)$).

We assume that when a node sends out a message, all neighboring nodes will receive this message immediately.

Algorithm 1: Localized Unit Delaunay Triangulation

1. Each wireless node u broadcasts an **announce** message with its identity and location to its neighbors $N_1(u)$ and listens to the messages from other nodes in $N_1(u)$.

2. Assume that node u gathered the location information of $N_1(u)$. It computes the Delaunay triangulation $Del(N_1(u))$ of its 1-neighbors $N_1(u)$, including u itself.

3. For each edge uv of $Del(N_1(u))$, let $\triangle uvw$ and $\triangle uvz$ be two triangles incident on uv. Edge uv is a Gabriel edge if both angles $\angle uwv$ and $\angle uzv$ are less than $\pi/2$. Node u marks all *Gabriel edges* uv, which will never be deleted.

4. Each node u finds all triangles $\triangle uvw$ from $Del(N_1(u))$ such that $\|vw\| \leq 1$ and $\angle wuv \geq \frac{\pi}{3}$. The node sorts the edges $uv \in Del(N_1(u))$ in clockwise order and broadcast a **proposal** message. The proposal message contains u's ID, followed by the IDs of the ordered nodes v with $uv \in Del(N_1(u))$. A single bit sent in between two IDs of v and w indicates if $\triangle uvw$ is as above, and we say that u is proposing that $\triangle uvw$ be added to $LDel^{(1)}(V)$. Then, node u listens to the messages from other nodes in $N_1(u)$.

5. Each node u, after receiving a proposal message from node v, computes if v is proposing adding triangles $\triangle uvw$ and $\triangle uvz$ to $LDel^{(1)}(V)$. Then, u computes if any of the triangles proposed by other nodes belongs to $Del(N_1(u))$. After computing all the triangles, u broadcast an **accept** message which contains the IDs of u, the clockwise list of vertices v such that u computed that for some w the triangle $\triangle uvw$ belongs to $Del(N_1(u))$, and a bit indicating if two such consecutive v, v' form a triangle $\triangle uvv'$ of $Del(N_1(u))$.

6. A node u adds the edges uv and uw to its set of incident edges if the triangle $\triangle uvw$ is in $Del(N_1(u))$ and both v and w have sent either a propose message or an accept message for triangle $\triangle uvw$.

   It turns out (see below) that the edges added by all the vertices in this step form $LDel^{(1)}(V)$. At this moment, the planarization phase starts.

7. Each node u broadcasts a **check** message with its ID and the position of all vertices v such that $uv \in LDel^{(1)}(V)$.

8. Based on the check messages sent by the nodes in $N_1(u)$, u computes for every 1-local Delaunay triangle $\triangle uvw$ if any node is inside the circumcircle of $\triangle uvw$. If such a node is found, u discards the triangle $\triangle uvw$. Then, the node u sorts clockwise the edges uv which are either Gabriel or belong to a triangle of $LDel^{(1)}(V)$ which was not discarded.

9. Each node u broadcasts an **alive** message, consisting of its ID, followed by the IDs of nodes incident to u and a single bit in between two such nodes v and w telling if $\triangle uvw \in LDel^{(1)}(V)$ and $\triangle uvw$ was not discarded.

10. Node u keeps the edge uv in its set of incident edges if it is a Gabriel edge, or if there is a triangle $\triangle uvw \in LDel^{(1)}(V)$ which appears in the alive messages of u, v, and w. This implicitly creates the graph $PLDel(V)$ and finishes the algorithm.

We first claim that the graph constructed at the end of Step 6 of the above algorithm is $LDel^{(1)}(V)$. Indeed, for each triangle $\triangle uvw$ of $LDel^{(1)}(V)$, one of its interior angle is at least $\pi/3$ and $\triangle uvw$ is in $Del(N_1(u))$, $Del(N_1(v))$, and

$Del(N_1(w))$. So, one of the nodes among $\{u, v, w\}$ will broadcast the message proposal to form a 1-localized Delaunay triangle $\triangle uvw$ and the other two nodes will accept the proposal. Thus, $LDel^{(1)}(V)$ is a subgraph of the constructed graph. Obviously, the constructed graph is also a subgraph of $LDel^{(1)}(V)$ by definition, which in turn implies that they are the same.

Second, we show that $PLDel(V)$ is indeed a planar graph.

Theorem 4. $PLDel(V)$ *is a planar graph.*

Proof. Two Gabriel edges do not intersect. Then, every intersection must involve a localized Delaunay triangle $\triangle xyz$ which was broadcast alive by all three x, y, and z.

Assume that an edge uw intersects a 1-localized Delaunay triangle $\triangle xyz$ on an edge xy. Edge uw is either a Gabriel edge or an edge of another 1-localized Delaunay triangle, say $\triangle uvw$. In either case, either Lemma 2 or Lemma 3 implies that either u or w is inside the $disk(x, y, z)$. By symmetry, we assume w is inside the $disk(x, y, z)$. The triangle inequality implies that

$$\|ux\| + \|wy\| < \|uw\| + \|xy\| \le 2.$$

The fact that triangle $\triangle xyz$ is in $PLDel(V)$ implies that $w \notin N_1(x) \cup N_1(y) \cup N_1(z)$. Thus, $\|wy\| > 1$, which implies that $\|ux\| < 1$. In other words, $u \in N_1(x)$.

When u broadcasts its check message, x does find out the existence of a node inside $disk(x, y, z)$. Then, x does not broadcast $\triangle xyz$ in its alive message. We obtained a contradiction, thus completing the proof. $\square$

Note that any triangle of $LDel^{(1)}(V)$ not kept by the algorithm is not a triangle of $LDel^{(2)}(V)$. Therefore, we have:

Theorem 5. $PLDel(V)$ *is a supergraph of* $LDel^{(2)}(V)$.

Next, we carefully analyze the communication cost of the algorithm. There are five types of messages and each includes a few bits to describe which type. Disregarding the few other bits used by every message, we have:

1. **announce** messages contain in total $n(p + q)$ bits, and
2. **proposal** messages contain in total $11nq$ bits. Indeed, if the proposal of a node includes the nodes $v_1, v_2, \ldots, v_j$, then for every $1 \le i \le j$, either $\angle v_i uv_{i+1} \ge \frac{\pi}{3}$ or $\angle v_{i-1} uv_i \ge \frac{\pi}{3}$ (where, for convenience, we assume $v_0 = v_j$ and $v_1 = v_{j+1}$). Immediate geometric arguments imply that $j \le 10$.
3. Each **accept** message contains the triangle incident with some vertex u. Those triangles are either "big" or "small" as defined in the proof of Theorem 2. The graph given by big triangles and the graph given by small triangles are planar. Using Euler's formula, we obtain that there are at most $4n$ triangles accepted by two nodes. In total, at most $6n$ triangles are implicitly proposed. Counting multiplicity (some proposed triangles are accepted once and some are accepted twice), a total of $10n$ triangles are accepted—each contributing one ID to the communication cost. As each accept message also contains the ID of the sender,

a total of at most $11n$ IDs are broadcast by accept messages.
4. Each **check** message contains the ID of a node u and a number of positions corresponding to u's neighbors in $LDel^{(1)}(V)$. As $LDel^{(1)}(V)$ has a thickness of two, it has at most $6n$ edges and, therefore, a total of at most $12n$ positions are broadcast in check messages.
5. Each **alive** message contains the ID of a node u and a number IDs corresponding to u's neighbors in $LDel^{(1)}(V)$, and bits indicating whether consecutive neighbors make a triangle not yet discarded. As shown above, there are at most $n + 12n = 13n$ IDs broadcast.

From the previous discussion, Theorem 1, Theorem 2, and the results of [16], [17] regarding the spanning ratio of $UDel(V)$, we obtain:

Theorem 6. $PLDel(V)$ *is planar* $\frac{4\sqrt{3}}{9}\pi$-*spanner of* $UDG(V)$, *and can be constructed with total communication cost of* $n(37q + 13p + 100)$ *bits, where* q *is the number of bits required to represent a node ID,* p *is the number of bits required to represent the position of a node, and* 100 *bits are used to represent all messages types.*

Recently, Calinescu [35] presented a localized method such that all wireless nodes collectively find the 2-hop neighbors $N_2(u)$ for every node u with $O(n \log n)$ communication complexity with the assumption that the geometric location of every node is known. The knowledge of the 2-hop neighbors information allows the direct construction of $LDel^2(V)$. However, the hidden constant of [35] is much larger than the constant presented here.

5 ROUTING

In this section, we discuss how to route packets on the constructed graph. Recently, Bose and Morin [9] first proposed a localized routing algorithm that routes a packet using the Delaunay triangulation and guarantees the distance traveled by the packet is no more than a small constant factor of the distance between the source and the destination nodes. However, this algorithm has a major deficiency by requiring the construction of the Delaunay triangulation and the Voronoi diagram of all wireless nodes, which could be very expensive in distributed computing. Bose et al. [11] also proposed another method, called Face routing, that routes the packets using the Gabriel graph to guarantee the delivery. The Gabriel graph is a subgraph of $PLDel(V)$. Thus, if we apply the routing method proposed in [11] on the newly proposed planar graph $PLDel(V)$, we expect to achieve better performance because $PLDel(V)$ is denser than the Gabriel graph (but, still with $O(n)$ edges). The constructed local Delaunay triangulation not only guarantees that the length of the shortest path connecting any two wireless nodes is at most a constant factor of the minimum in the unit-disk graph, but it also guarantees that the energy consumed by the path is also minimum, as it includes the Gabriel graph (see [36], [27]). Moreover, because the constructed topology is planar, a localized routing algorithm using the right-hand rule guarantees the delivery of the packets from source node to the destination node.
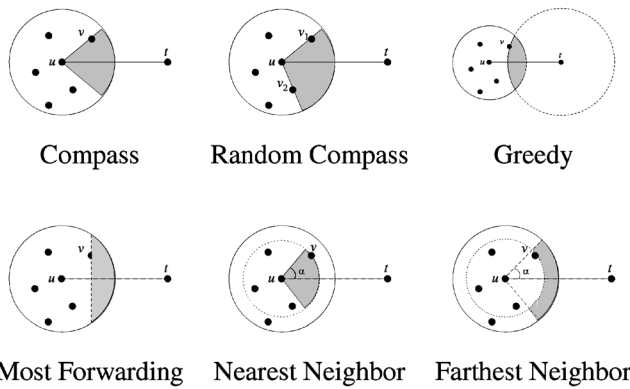
Fig. 3. Shaded area is empty and v is next node.

We study the following routing algorithms, illustrated by Fig. 3, on the graphs proposed in this paper.

Compass Routing (Cmp) Let t be the destination node. Current node u finds the next relay node v such that the angle $\angle vut$ is the smallest among all neighbors of u in a given topology. See [20].

Random Compass Routing (RndCmp) Let u be the current node and t be the destination node. Let v_1 be the node on the above of line ut such that $\angle v_1 ut$ is the smallest among all such neighbors of u. Similarly, we define v_2 to be nodes below line ut that minimizes the angle $\angle v_2 ut$. Then, node u randomly chooses v_1 or v_2 to forward the packet. See [20].

Greedy Routing (Grdy) Let t be the destination node. Current node u finds the next relay node v such that the distance $\|vt\|$ is the smallest among all neighbors of u in a given topology. See [11].

Most Forwarding Routing (MFR) Current node u finds the next relay node v such that $\|v't\|$ is the smallest among all neighbors of u in a given topology, where v' is the projection of v on segment ut. See [10].

Nearest Neighbor Routing (NN) Given a parameter angle α, node u finds the nearest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$.

Farthest Neighbor Routing (FN) Given a parameter angle α, node u finds the farthest node v as forwarding node among all neighbors of u in a given topology such that $\angle vut \leq \alpha$.

It is shown in [11], [20] that the compass routing, random compass routing, and the greedy routing guarantee to deliver the packets from the source to the destination if Delaunay triangulation is used as network topology. They proved this by showing that the distance from the selected forwarding node v to the destination node t is less than the distance from current node u to t. However, the same proof cannot be carried over when the network topology is Yao graph, Gabriel graph, relative neighborhood graph, and the localized Delaunay triangulation.

We present our experimental results of various routing methods on different network topologies. Fig. 4 illustrates some network topologies discussed in this paper. Recall that Gabriel graph, relative neighborhood graph, Delaunay triangulation, $LDel^{(2)}(V)$, and $PLDel(V)$ are always planar graphs. The Yao structure, Delaunay triangulation, $LDel^{(2)}(V)$, and $PLDel(V)$ are always a t-spanner of the unit-disk graph. We use integer parameter $k = 8$ in constructing the Yao graph. In the experimental results presented here, we choose total $n = 50$ wireless nodes which are distributed randomly in a square area with side length 100 meters. Each node is specified by a random x-coordinate value and a random y-coordinate value. The transmission radius of each wireless node is set as 30 meters. We randomly select 10 percent of nodes as source nodes and, for every source node, we randomly choose 10 percent of nodes as destination nodes. The statistics are computed over 10 different node configurations. Interestingly, we found that, when the underlying network topology is Yao graph, $LDel^{(2)}(V)$, or $PLDel(V)$, the compass routing, random compass routing, and the greedy routing delivered the packets in all our experiments. Table 1 illustrates the delivery rates of different localized routing protocols on various topologies. For nearest neighbor routing and farthest neighbor routing, we choose the angle $\alpha = \pi/3$. The $LDel^{(2)}(V)$ and $PLDel(V)$ graphs be preferred over the Yao graph because we can apply the right-hand rule when previous simple heuristic localized routing fails. The reason that the compass, random compass, and greedy methods are able to guarantee the delivery of the packets in our simulations may be as follows: When the transmission range of a node is large enough, the localized Delaunay triangulation of a randomly uniformly distributed point set is almost the same as the Delaunay triangulation (this can be proven [37]). We already know that these three methods guarantee delivery if Delaunay triangulation is used. RNG and GG have small delivery rate when these simple localized heuristics are used since these two structures have less edges than other structures and, therefore, each node often has less choices of nodes to relay messages. Table 2 illustrates the maximum spanning ratios achieved by different routing methods on various topologies. In our experiment, these ratios are considerably small.

We also conducted extensive simulations of the Face routing method [11] and the Greedy-Face-Greedy (GFG)
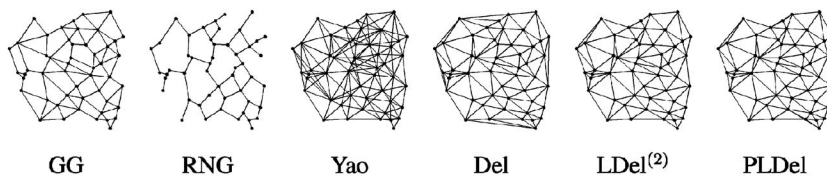

Fig. 4. Various planar network topologies (except Yao).

TABLE 1
The Delivery Rate of Different Localized Routing Methods on Various Network Topologies

	Yao	RNG	GG	Del	LDel[(2)]	PLDel
NN	98.7	44.9	83.2	99.1	97.8	98.3
FN	97.5	49	81.7	92.1	97	97.6
MFR	98.5	78.5	96.6	95.2	96.6	99.7
Cmp	100	86.6	99.6	100	100	100
RndCmp	100	91.7	99.9	100	100	100
Grdy	100	87.5	99.6	100	100	100

TABLE 2
The Maximum Spanning Ratio of Different Localized Routing Methods on Various Network Topologies

	Yao	RNG	GG	Del	LDel[(2)]	PLDel
NN	1.9	2.1	1.9	1.7	1.8	1.9
FN	4.2	2.8	2.7	5.2	3.4	3.1
MFR	4.8	3.2	2.4	4.5	3.9	4.1
Cmp	3.3	2.9	2.8	1.6	1.8	2.0
RndCmp	2.7	3.0	2.4	1.7	2.0	1.8
Grdy	2.1	3.5	2.2	2.0	1.9	1.9

routing method [38] on the Gabriel graph and the local Delaunay triangulation $LDel^1(V)$. We choose $n = 20, 30, \cdots, 90, 100$ nodes randomly and uniformly distributed in a square of length 100 meters. The uniform transmission range of nodes are set as r, where r varies from $30, 40, 50, 60,$ or 70 meters. Since it will be hard to control the network density directly, we change the network density by indirectly changing the number of nodes in the network or the transmission range of the nodes. A large number of nodes or larger transmission range will result in a denser network. See Tables 3 and 4. For a given point set, we randomly select 10 percent of nodes as sources and 10 percent of nodes as targets. The maximum and average spanning ratio is computed for all chosen pair of nodes. Given n and r, we generate 10 sets of random n points. We found that the spanning ratio of the Face routing method is significantly smaller when the local Delaunay triangulation is used instead of the Gabriel graph. This may be due to the fact that local Delaunay triangulation has more edges, thus the number of faces traversed by the Face routing algorithm is often smaller when LDEL is used than when GG is used.

The average spanning ratios of the GFG method on the Gabriel graph and the local Delaunay triangulation are

similar when the number of nodes in the network is no more than 100. We expect our local Delaunay triangulation to perform better when the number of nodes are significantly large since, the spanning ratio of the Gabriel graph on a set of n random points is about $\Omega(\sqrt{\log n / \log\log n})$. When n is small, $\Omega(\sqrt{\log n / \log\log n})$ is almost a small constant, but when n is large, $\Omega(\sqrt{\log n / \log\log n})$ cannot be treated as a small constant anymore. Notice that our local Delaunay triangulation guarantees that there is always a path to connect any two nodes with length no more than 2.5 times the length of the shortest path connecting them in the original UDG.

6 CONCLUSION

It is well-known that Delaunay triangulation $Del(V)$ is a t-spanner of the completed graph $K(V)$. In this paper, we defined several new structures and then gave a localized algorithm that constructs a graph, namely, $PLDel(V)$. We proved that $PLDel(V)$ is a planar graph and it is a t-spanner by showing that $UDel(V)$ is a subgraph of $PLDel(V)$. The total number of messages sent by all nodes in our algorithm is $O(n \log n)$ bits. Our experiments showed that the delivery

TABLE 3
The Maximum Spanning Ratio of Face (over the Fraction Line) and GFG (under the Fraction Line) Routing Methods on the Gabriel Graph and Local Delaunay Triangulation

n	0.3	0.4	0.5	0.6	0.7
20	11.5/7.6	15.4/4.9	9.9/1.7	18.4/1.6	13.0/1.6
30	34.2/5.8	17.9/5.6	18.3/2.0	14.7/2.0	21.5/3.1
40	22.7/16.6	21.1/9.3	14.7/9.8	16.2/4.6	15.6/13.8
50	15.4/1.9	20.6/1.8	31.5/1.9	14.6/1.6	18.0/2.0
60	27.6/7.6	17.3/1.6	18.5/1.7	19.1/14.2	25.7/1.8
70	25.3/17.7	30.4/9.9	19.5/1.9	22.0/2.6	20.9/1.9
80	30.1/1.7	49.4/1.6	22.4/1.7	20.8/6.5	23.4/1.7
90	16.0/1.9	33.7/2.8	14.9/3.7	32.2/10.4	19.8/1.9
100	19.2/2.5	43.8/3.5	13.5/2.9	54.6/3.9	24.6/2.7

On Gabriel graph

n	0.3	0.4	0.5	0.6	0.7
20	10.6/5.5	7.8/1.6	9.6/3.7	13.2/1.6	7.7/1.6
30	10.3/6.6	14.2/4.5	33.3/4.1	14.2/2.2	13.9/2.1
40	18.0/12.3	24.1/2.2	21.4/2.6	30.5/10.3	14.9/1.8
50	11.3/2.3	21.6/1.8	14.7/1.5	23.2/2.1	20.1/2.3
60	18.3/2.8	21.9/1.9	22.0/1.7	16.2/2.2	17.4/2.8
70	23.0/1.6	60.2/1.8	31.0/1.8	19.0/1.9	22.8/1.8
80	24.7/11.2	24.4/2.2	20.6/2.8	33.9/2.2	13.6/5.4
90	31.5/1.9	20.1/2.2	20.2/3.0	52.8/2.0	50.4/1.8
100	24.9/2.6	11.1/4.4	17.2/2.2	16.7/9.2	24.8/2.9

On local Delaunay triangulation

TABLE 4
The Average Spanning Ratio of Face (over the Fraction Line) and GFG (under the Fraction Line) Routing Methods
on the Gabriel Graph and Local Delaunay Triangulation

n	0.3	0.4	0.5	0.6	0.7
20	3.0/1.8	2.7/1.2	2.7/1.2	3.0/1.1	2.4/1.2
30	3.4/1.3	3.2/1.2	3.7/1.2	3.3/1.2	3.6/1.2
40	4.2/1.2	3.8/1.2	3.7/1.3	3.6/1.2	3.7/1.3
50	4.4/1.2	4.3/1.2	4.6/1.2	3.8/1.2	4.0/1.2
60	4.6/1.2	4.4/1.2	4.5/1.2	4.6/1.3	4.6/1.2
70	5.2/1.2	4.9/1.2	4.9/1.2	5.0/1.2	5.0/1.2
80	5.1/1.2	5.5/1.2	5.7/1.2	5.1/1.2	5.3/1.2
90	5.0/1.2	4.9/1.2	4.7/1.3	5.2/1.2	5.5/1.2
100	5.2/1.2	5.8/1.2	6.3/1.2	5.8/1.3	5.6/1.2

On Gabriel graph

n	0.3	0.4	0.5	0.6	0.7
20	2.9/1.3	2.5/1.1	3.0/1.1	2.6/1.1	2.2/1.1
30	3.2/1.2	2.9/1.2	3.5/1.2	2.9/1.2	3.4/1.2
40	3.9/1.2	3.8/1.2	4.2/1.2	3.5/1.2	4.2/1.2
50	3.3/1.2	4.0/1.2	4.2/1.2	3.9/1.1	4.0/1.2
60	4.4/1.2	5.0/1.2	4.7/1.2	4.4/1.2	4.4/1.2
70	4.9/1.1	5.3/1.2	5.5/1.2	4.9/1.2	5.0/1.2
80	5.2/1.2	5.4/1.2	5.1/1.2	5.4/1.2	5.0/1.2
90	5.5/1.2	5.9/1.2	5.6/1.2	5.6/1.2	5.3/1.2
100	6.1/1.3	5.2/1.2	6.0/1.2	6.0/1.4	6.4/1.2

On local Delaunay triangulation

rates of existing localized routing protocols are increased when localized Delaunay triangulation is used instead of several previously proposed planar topologies. Our simulations also shows that the traveled distance of the packets is significantly less when the FACE routing algorithm or GFG routing is applied on LDEL than applied on GG.

We proved that the shortest path in $PLDel(V)$ connecting any two nodes u and v is at most a constant factor of the shortest path connecting u and v in UDG. It remains open designing a localized algorithm such that the path traversed by a packet from u to v has length within a constant of the shortest path connecting u and v in UDG.

Recently, Li et al. [39], [40] present a novel method to construct bounded degree planar spanner locally.

APPENDIX

We start this with the proof of Lemma 3, which we restate below:

Lemma 4. *Let $\triangle uvw$ and $\triangle xyz$ be two triangles of $LDel^{(k)}(V)$, $k \geq 1$, and assume the edge xy intersects the triangle $\triangle uvw$ and that $disk(x, y, z)$ does not contain any of the nodes of $\{u, v, w\}$. Then, $disk(u, v, w)$ contains either x or y.*

The following lemma is implicitly used in its proof.

Lemma 5. *If an edge xy intersects a localized Delaunay triangle $\triangle uvw$, then it intersects two edges of $\triangle uvw$.*

Proof. If it intersects one edge of $\triangle uvw$, then either x or y must be inside $\triangle uvw$, say x. Then $xu < \max(uv, uw) \leq 1$, which contradicts that $\triangle uvw$ is a localized Delaunay triangle. □

We present some other useful facts and lemmas.

Lemma 6. *If an edge xy intersects a localized Delaunay triangle $\triangle uvw$, then x and y can not be both inside the circumcircle $disk(u, v, w)$.*

Proof. For the sake of contradiction, assume that x and y are both inside $disk(u, v, w)$. Fig. 5 illustrates the proof that follows. Notice that $disk(u, v, w)$ is divided into four regions by the triangle $\triangle uvw$. Let $\widehat{uv}$, $\widehat{vw}$, and $\widehat{wu}$ be the three fan regions defined by edges uv, vw, and wu, respectively. First of all, neither x nor y can be inside the triangle $\triangle uvw$. Assume that x is inside the region $\widehat{uv}$ and y is inside the region $\widehat{vw}$. Then, one of the angles $\angle uwv$ and $\angle vuw$ is less than $\frac{\pi}{2}$, which implies that one of the angles $\angle uxv$ and $\angle vyw$ is larger than $\frac{\pi}{2}$. Thus, either $vy < vw \leq 1$ or $vx < vu \leq 1$. In other words, the $disk(u, v, w)$ contains a node from $N_1(v)$. This contradicts that $\triangle uvw$ is a k-localized Delaunay triangle. □

We now present the proof of Lemma 3.

Proof. There are three cases: triangles $\triangle uvw$ and $\triangle xyz$ share two nodes (i.e., one edge), one node, or do not share any node.

Case 1: Triangles $\triangle uvw$ and $\triangle xyz$ share one edge.

Case 2: Triangles $\triangle uvw$ and $\triangle xyz$ share one node. These two cases are impossible. See the proof of
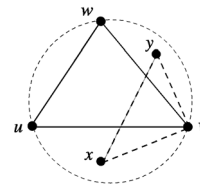

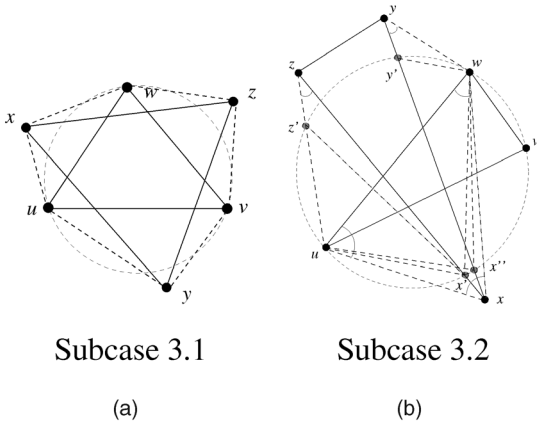Fig. 5. Points x, y cannot both be inside $disk(u, v, w)$.

Subcase 3.1   Subcase 3.2

(a)   (b)

Fig. 6. All or four edges of two triangles intersect.

(a)   (b)

Fig. 7. Subcase 3.1 is impossible.

Theorem 10 in the Appendix of our conference version [16].

Case 3: Triangles $\triangle uvw$ and $\triangle xyz$ do not share any node. It is not difficult to show that there are only two possible subcases as illustrated by Fig. 6.

Subcase 3.1: All edges of $\triangle xyz$ and $\triangle uvw$ are intersected by some edges of the other triangle. Assume that the nodes have the order as illustrated by the Fig. 6a. Then, it is easy to show that all angles $\angle wxu$, $\angle xuy$, $\angle uyv$, $\angle yvz$, $\angle vzw$, and $\angle zwx$ are less than π. Notice that $\angle wxu + \angle wvu < \pi$ because x is not inside the circumcircle $disk(u,v,w)$. Similarly, $\angle uyv + \angle uwv < \pi$ and $\angle vzw + \angle vuw < \pi$. Therefore,

$$\angle wxu + \angle uyv + \angle vzw < 3\pi - (\angle wvu + \angle uwv + \angle vuw) = 2\pi.$$

Notice that

$$\angle wxu + \angle uyv + \angle vzw + \angle xuy + \angle yvz + \angle zwx = 4\pi.$$

It implies that $\angle xuy + \angle yvz + \angle zwx > 2\pi$. Then, we know that at least one of the nodes of u, v, and w is contained inside the circumcircle $disk(x,y,z)$ (otherwise by symmetry, similarly, we would have $\angle xuy + \angle yvz + \angle zwx < 2\pi$). We prove that Subcase 3.1 is impossible. For the sake of contradiction, assume that it is possible. Then, from the proof of Subcase 3.1, either $disk(u,v,w)$ contains one of the nodes of x, y, and z; or $disk(x,y,z)$ contains at least one of the nodes of u, v, and w. Without loss of generality, assume that node x is contained in the interior of $disk(u,v,w)$. Then, Lemma 6 implies that both y and z are outside of $disk(u,v,w)$. Fig. 7 illustrates the proof that follows. The existence of triangle $\triangle uvw$ implies that $\|xu\| > 1$, $\|xv\| > 1$, and $\|xw\| > 1$. Notice that $\|xy\| \le 1$ and $\|xz\| \le 1$. Let c be the circumcenter of the triangle $\triangle uvw$. Here, c cannot be x because $xu > 1$, $xy \le 1$, and y is outside of the circle. The angle $\angle uxv < \frac{\pi}{3}$ because uv must be the shortest edge of triangle $\triangle uxv$. Consider the following five segments lying in the interior of the wedge uxv: xv, xz, xw, xy, and xu. From the pigeonhole principle, there are at least three such segments lying on the same side of the line xc. More precisely, we have either xv, xz, and xw on the same side of xc or xw, xy, and xu are on the same side of xc. Without loss of generality, assume that the first
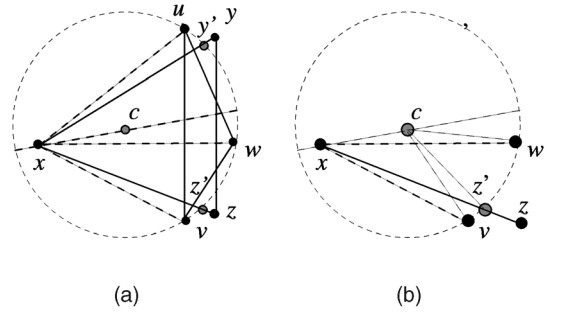
scenario happens. Then, it is easy to prove that $\|xz\| > \min(xv, xw) > 1$. This contradicts to $\|xz\| \le 1$. Fig. 7b illustrates the proof using that

$$\|xv\|^2 = \|xc\|^2 + \|cv\|^2 - 2\|xc\| \cdot \|cv\| \cdot \cos(\angle xcv),$$

and $\|cv\| = \|cz'\| = \|cw\|$. Therefore, the assumption that Subcase 3.1 is possible does not hold.

Subcase 3.2: One edge of each triangle is not intersected by the edges of the other triangle. We prove that $disk(x,y,z)$ contains at least one of the nodes of u and w. Fig. 6b illustrates the proof that follows. Let x' be the intersection point of segment xz with the circumcircle $disk(u,v,w)$, which is closer to x. Let z' be the intersection point of segment uz with the circumcircle $disk(u,v,w)$. Let x'' and y' be the two intersection points of segment xy with the circumcircle $disk(u,v,w)$, where x'' is closer to x and y' is closer to y. Then,

$$\angle xzu < \angle x'z'u = \angle x'wu < \angle xwu, \text{ and}$$
$$\angle wyx < \angle wy'x'' = \angle wux'' < \angle wux.$$

Notice that $\angle yzu + \angle zux + \angle uxw + \angle xwy + \angle wyz = 3\pi$. Then,

$$(\angle yzx + \angle ywx) + (\angle zyx + \angle zux) = 3\pi - (\angle xzu + \angle wyx + \angle uxw)$$
$$> 3\pi - (\angle xwu + \angle wux + \angle uxw) = 2\pi.$$

So, either $\angle yzx + \angle ywx > \pi$ or $\angle zyx + \angle zux > \pi$ from the pigeonhole principle. Consequently, $disk(x,y,z)$ contains either node w or node u. $\square$

We present the proof that graph $LDel^{(1)}(V)$ has a thickness of two.

Proof. We prove that $LDel^{(1)}(V)$ has thickness two or, in other words, its edges can be partitioned in two planar graphs. Call a triangle $\triangle xyz$ of $LDel^{(1)}(V)$ *big* if the radius of $disk(u,v,w)$ is at least one, and *small* otherwise. A Gabriel edge can be thought as a small triangle, as if one of its endpoints is duplicated. Put all the Gabriel edges and the edges of small triangles in the graph G_1 and all the edges of the big triangles in G_2, possibly putting an edge in both G_1 and G_2 if it is shared by a small triangle and a big triangle. In the following, we show that both G_1 and G_2 are planar. More precisely, we prove that any intersection of two edges of $LDel^{(1)}(V)$ involves a big triangle and a small triangle, or a big triangle and a Gabriel edge.

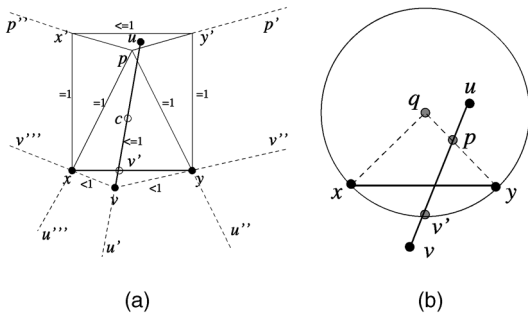First, we show that the edges of two big triangles cannot intersect. Assume for a contradiction that $\triangle uvw$

Fig. 8. (a) No two big triangles intersect. (b) No two small triangles intersect.

and $\triangle xyz$ are big triangles which intersect. We must be in Subcase 3.2 of the proof of Lemma 3. Rename the vertices of the triangles such that the following situation holds: u is inside $disk(x, y, z)$, v is outside $disk(x, y, z)$, and the edges uv and xy intersect in the point v'. Then, $\|ux\| > 1$ and $\|uy\| > 1$. Using the triangle inequality in $\triangle xv'v$ and $\triangle uv'y$, we obtain that $\|vx\| < 1$ and, similarly, $\|vy\| < 1$. Then, $\|qu\| = \|qv\| \geq 1$. Let x' and y' be the two points such that $xx'y'y$ is a rectangle with $\|xx'\| = \|yy'\| = 1$ such that u and x' are on the same side of the line xy. Let p be the point inside $xx'y'y$ such that $\|px\| = \|py\| = 1$. Note that u must be inside the rectangle $xx'y'y$. Indeed, otherwise, $1 = \|xx'\| \leq \|uv'\| < 1$. As $\|uv'\| < 1$, u is inside the triangle $\triangle x'y'p$ or in the interior of the edge $x'y'$ because u cannot be inside the triangle $\triangle xx'p$ (otherwise, $\|ux\| \leq \max(\|x'x\|, \|px\|) = 1$, which is impossible); similarly, u cannot be inside the triangle $\triangle yy'p$ and the triangle $\triangle xyp$. Let q be the center of $disk(u, v, w)$. Let c be the midpoint of segment uv. Thus, cq is a perpendicular bisector of segment uv. Then, c must be between points u and v'. Indeed, otherwise, either x or y is on the same side of cq with u, which contradicts to $\|ux\| > 1 \geq \|vx\|$ and $\|uy\| > 1 \geq \|vy\|$. We show that there is no place to put the circumcenter q. Fig. 8 illustrates the proof that follows. The *closed* infinite area inside an angle $\angle p_1 p_2 p_3$, also referred to as a *sector*, is denoted by $\angle p_1 p_2 p_3$. In addition, sector $\angle p_1 p_2 p_3 p_4$ denotes the intersection of two sectors $\angle p_1 p_2 p_3$ and $p_2 p_3 p_4$. First, point q cannot be inside the sector $\angle u''yv''$. Otherwise, $\|qy\| < \max(\|qu\|, \|qv\|)$ because y is inside the triangle $\triangle quv$. Thus, y is inside $disk(u, v, w)$. Then, triangle $\triangle uvw$ should not exist, as v can see y inside $disk(u, v, w)$. Symmetry implies that also q cannot be inside the sector $\angle u'''xv'''$. Second, point q cannot be inside the sector $\angle uvyu''$. Otherwise, segment qc intersects yv, which implies that $\|vy\| > \|uy\| > 1$. It contradicts to $1 \geq \|vy\|$. Similarly, point q also cannot be inside the sector $u'''xvu'$. Third, point q cannot be inside the triangle $\triangle xyv$. Otherwise, $\|qv\| \leq \max(\|vx\|, \|vy\|) < 1$. It contradicts to that triangle $\triangle uvw$ is big. Fourth, point q cannot be inside the sector $\angle p'x'xv'''$. Indeed, the segment qv' cannot intersect the segment xx'. Otherwise, $\|qx\| \leq \|qv'\| \leq \|qv\|$ and the triangle $\triangle uvw$ should not exist, as v can see x inside $disk(u, v, w)$. Symmetry also implies that point

q cannot be inside the sector $\angle v''yy'p'$. Fifth, q cannot be inside the sector $\angle p'y'x'p''$ and the segment qv' intersects the segment $x'x$. Otherwise, let q' be the intersection point of qv' and $x'x$. Since the biggest edge of $\triangle x'y'p$ has length 1, $\|qu\| < 1$. Then, from $\|q'v'\| \geq \|xx'\| = 1$, we have

$$\|qu\| \leq \|qq'\| + \|q'u\| < \|qq'\| + 1 \leq \|qq'\| + \|q'v'\|$$
$$= \|qv'\| \leq \|qv\|,$$

contradicting the fact that $\|qu\| = \|qv\|$. Thus, q must be inside the rectangle $xx'y'y$. Note $\|qx\| > \|qu\| \geq 1$ and $\|qy\| > \|qu\| \geq 1$. Therefore, if q is inside the rectangle $xx'y'y$, it is inside the triangle $\triangle x'y'p$. Since the biggest edge of $\triangle x'y'p$ has length 1, $\|qu\| < 1$, a contradiction. In conclusion, two big triangles cannot intersect.

Now, we show that the edges of two small triangles cannot intersect, and that the edges of a small triangle cannot intersect a Gabriel edge. Fig. 8 illustrates the proof that follows. As shown above, we can assume that u is inside $disk(x, y, z)$ and v is outside $disk(x, y, z)$. Moreover, $\|ux\| > 1$ and $\|uy\| > 1$. Let v' be the intersection of the segment uv with the boundary of $disk(x, y, z)$. Let q be the center of $disk(x, y, z)$. Since $\|qy\| = \|qx\| < 1$, u cannot be inside the triangle $\triangle qxy$ and, therefore, the segment uv' intersects either the segment xq or the segment yq. By symmetry, we assume that uv' and qy intersect in point p. We have that $\|qv'\| = \|qy\|$ and that $\|uy\| > 1$. Using triangle inequality for $\triangle qpv'$ and $\triangle upq$, we have $\|qy\| + \|uv'\| > \|qv'\| + \|yu\|$. It deduces that $\|uv'\| > 1$ and, therefore $\|uv\| > 1$, a contradiction. This completes the proof of the theorem. ☐

ACKNOWLEDGMENTS

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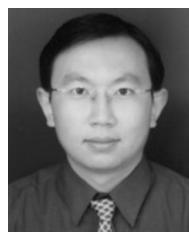
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