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# Truthful Online Spectrum Allocation and Auction in Multi-Channel Wireless Networks

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***Abstract*—We propose efficient spectrum channel allocation and auction methods for the online wireless channel scheduling. Assume that each user requests for the exclusive usage of a number of wireless channels for a certain time interval. The scheduler has to decide whether to grant its exclusive usage an how much will be charged. To possibly serve users with higher priority, preemptions are allowed with penalties. We analytically prove that our protocols are efficient, truthful, and they have asymptotically optimum competitive ratios. Our extensive simulations show that they perform almost optimum: most of our methods can achieve more than $50\%$ of the optimum by offline method.**

***Index Terms*—Spectrum, online algorithm, competitive ratio, wireless networks, mechanisms, strategyproof.**

## I. Introduction

With the recent fast growing of spectrum-based services and devices, the remaining spectrum available for future wireless services is being exhausted, known as the *spectrum scarcity*. The current fixed spectrum allocation scheme leads to significant spectrum *white spaces* (including spectral, temporal, and geographic). Among methods proposed to improve spectrum usage, the cognitive radio approaches [11], [17]–[19] and the market-driven auctions between primary users and secondary users [4], [5], [7], [12], [21], [27], [28] have attracted considerable research interests recently.

Subleasing is another potential way to share spectrum. Our approach is different from the FCC-style spectrum auctions that typically target long term leases of spectrum in a large area. Here, as in [27], [28], we consider a dynamic spectrum channel allocation and auction system that serves many small players without manual negotiations.

Most recent studies [16], [27], [28] assume that all information is known before dynamic spectrum allocation. This is not true generally: the requests often arrive online and the central authority needs to quickly make a decision without further knowledge. Assume there is a sequence of requests arrive online. Our online algorithm, upon receiving a request, needs to make a decision immediately. Cancelation is necessary for channel owner to take advantage of a spike in demand and rising prices for channels and not be forced to sell the spectrum-slots below the market because of an a priori contract. The channel owner benefits from the reduction in uncertainty, and pays for this via penalties.

In this work, the performance of an online method $\mathcal{A}$ is measured by its *competitive ratio* $\varrho(\mathcal{A}) = \min_{\mathcal{I}} \frac{\mathcal{A}(\mathcal{I})}{\mathrm{OPT}(\mathcal{I})}$, where the optimum *offline method* knows the sequence $\mathcal{I}$ in advance. The main contributions of this paper are as follows:

We show, in Theorem 1, to guarantee a competitive ratio, we have to utilize some additional information. As an example, we will focus on the scenario that the number of time-slots requests by each bid is at most $\Delta$. Theorem 3 show that **no** online method will have a competitive ratio $> \sqrt{2}\Delta^{-\frac{1}{2}}$.

In Section IV, we propose an efficient Algorithm 1, whose competitive ratio is at least $\frac{3\sqrt{2}-4}{16}\Delta^{-\frac{1}{2}}$. We also design an auction mechanism. Our extensive simulations show that our mechanisms indeed get a revenue that is close to optimum. In most cases, our methods are able to get a revenue that is $> 50\%$ of the optimum offline method.

The rest of paper is organized as follows. Section II presents our network model and questions to be studied. In Section III, we present upper bounds on the competitive ratios. We then present our solutions in Section IV and analytically prove the performance bounds of our methods. Then we extend our methods to deal with other interference models in Section V. In Section VI, we design auction mechanisms based on our online algorithm. Our simulation studies are reported in Section VII. We review the related work in Section VIII and conclude the paper in Section IX.

## II. Preliminaries

### A. Network Model

Assume that there are $m$ *identical* channels. which means all channels have homogeneous performances. And a central authority will decide the assignment of those spectrum channels to $n$ secondary users without any prediction.

Secondary users may reside at different geometry locations, which are modeled by a conflict graph $H = (V, E)$, where two nodes $v_i$ and $v_j$ (denoting two different secondary users) form an edge $(v_i, v_j)$ in $H$ iff they conflict with each other. We first address the case in which the conflict graph $H$ is a completed graph. Then we show that our methods can be easily extended to the case in which $H$ has a constant *one-hop independence number*.

Preemptions are allowed with penalties. In our model, penalty depends on the requests, preemption times and preemption models. Two different preemption models will be studied. In the first model, a request is terminated only if *all* usage its channels are preempted. In the second model (called *all-or-nothing* model), a request is terminated if *any* usage of its channels is preempted. We focus on the first model and show our method can be extended to the second model with asymptotically same competitive ratio.

### B. Problems Formulation

Assume there is a sequence of requests $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_i, \cdots$. Each request $\mathbf{r}_i = (b_i, k_i, a_i, s_i, t_i)$ arrives at time $a_i$, bids $b_i$ for the usage of $k_i$ channels from time $s_i$ to time $s_i + t_i$. Here $s_i = a_i$, which means central authority should make a decision on each request immediately. Time is slotted and all requirements are integers between $[1, \Delta]$.

When a request $\mathbf{r}_i$ is granted, an amount $b_i$ will be charged if all users are truthful, or a monetary value $\mathbf{p}_i$ using some mechanisms designed in Section VI when users are selfish. Preemptions are allowed with penalties, which are defined by $\gamma(b_i, k_i, k'_i, t_i, t'_i)$, where $k'_i \leq k_i$ is the number of channels which are taken back, $t'_i$ is the unfinished time. Two different preemption models will be studied. In the first model, a request is terminated only if *all* usage its channels are preempted, where $\gamma(b_i, k_i, k'_i, t_i, t'_i) = \frac{k'_i t'_i}{k_i t_i} b_i$. In the second model (called *all-or-nothing* model), a request is terminated if *any* usage of its channels is preempted, where $\gamma(b_i, k_i, k'_i, t_i, t'_i) = \frac{t'_i}{t_i} b_i$. We start from first model and show our method can be extended to second model with almost same competitive ratio.

The objective is to maximize the total net profit, *i.e.*, the total payments collected minus the total preemption penalties.

## III. Performance Upper Bound

In this section, we study the performance upper bound of *any* online method. Due to space limit, in this section and following sections, most proofs are omitted. To check the proofs, please see our technical report [24].

First, we show that the performance could be arbitrarily bad nothing is unknown. Then, we show the performance upper bound if time-ratio $\Delta$ is known. Typically, the competitive ratio of an online method is analyzed using the *adversary model*, where spectrum requests are generated by an adversary whose goal is to beat the designed algorithm. In other words, given *any* deterministic or random method, at any time slot, the adversary will *strategically* issue some spectrum requests based on the historical decisions made by the algorithm. By using adversary model, we have following theorem.

*Theorem 1:* There is no online algorithm with competitive ratio more than $\frac{1}{\rho}$ for any constant $\rho > 1$.

Thus, to achieve some reasonable bound on competitive ratio, some additional information must be known. Given the *time-ratio* $\Delta$, the performance upper bound is as follows.

*Theorem 2:* There is no online algorithm with competitive ratio more than $2\Delta^{-\frac{1}{3}}$ even we know the time-ratio $\Delta$.

*Theorem 3:* There is no online algorithm with competitive ratio more than $\frac{1}{\rho}$ for constant $\rho$ with $\frac{1}{2}\Delta^{\frac{1}{3}} \leq \rho < \frac{\sqrt{2}}{2}\Delta^{\frac{1}{2}}$.

Theorem 3 implies that the competitive ratio of *any* online spectrum allocation algorithm (when all users are truthful) is at most $(\sqrt{2}+\epsilon)\Delta^{-\frac{1}{2}}$ for an arbitrarily small constant $\epsilon > 0$.

## IV. Efficient Online Admission

For results presented in this section, we assume that the interference graph $H$ is a complete graph and the central authority can preempt *any* number of channels allocated to a user, not necessarily all allocated channels to a user.

### A. Our Method

The basic idea is that we always satisfy those requests with either large total bid or large bid per time slot (called value *density*). Notice we consider each empty channel is being occupied by a *virtual request* with bid 0. In our method and analysis, we will always process the running spectrum usage as from individual *virtual requests*.

*Definition 1 ($n$-Strong Preemption with Constant $c$):* Given requests set $\mathcal{R}_1$ at time $t$, we find a subset $\mathcal{R}'_1 \subset \mathcal{R}_1$ of conflict-free requests, say $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\} \subseteq \mathcal{R}_1$, such that (1) they do not conflict in geometry locations and in requested time intervals, and (2) the summation of their bids values $\sum_{j=1}^{i} b_j$ is maximized, *i.e.*, $\arg\max_{\{\mathbf{r}_1,\cdots,\mathbf{r}_i\}} \sum_{j=1}^{i} b_j$; and (3) $n$ is the total number of requested channels, *i.e.*, $n = \sum_{j=1}^{i} k_j$.

Let $\mathcal{R}_2$ be the requests set which currently occupy the channels. Since we apply individual channel preemption, each virtual running request $\mathbf{r}'_j \in \mathcal{R}_2$, created from some original request $\mathbf{r}_q$, has a bid value $b'_j = b_q/k_q$ now and $k'_j = 1$. If there exists a subset $\mathcal{R}'_2 \subset \mathcal{R}_2$ of virtual requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\} \subseteq \mathcal{R}_2$ such that

$$\begin{cases} \sum_{j=1}^{i} b_j \geq c \sum_{j=1}^{i'} b'_j, \text{ and} \\ \sum_{j=1}^{i'} k'_j = n \end{cases} \quad (1)$$

we preempt virtual requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\}$ with requests $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\}$. Here $c > 1$ is some predefined constant. We say requests $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\}$ *$n$-strongly preempt* (virtual) requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\}$.

Here $n$ is an integer in $[1, m]$. When for the found subset $\mathcal{R}'_1$, $\sum_{j=1}^{i'} b'_j$ is minimized in all feasible $n$-strong Preemptions, we call it *minimum $n$-strong preemption*. Clearly, finding the subset $\mathcal{R}'_1$ is essentially the *knapsack* problem, which admits a fully polynomial time approximation scheme (FPTAS) [2]. Finding the subset $\mathcal{R}'_2$ clearly can be solved using simple greedy approach: we repeatedly free a channel assigned to a currently running virtual request with the smallest value $b'_q$ (which is $b_q/k_q$) till $n$ channels have been freed. Thus, if there exist *$n$-strong preemptions*, we can find the approximated minimum one in polynomial time such that the total bid values $\sum_{j=1}^{i} b_j$ is at least $1 - \epsilon$ of the optimum one. Otherwise, we can also know that there is no *$n$-strong preemption* in polynomial time. Intuitively, $n$-strong preemption is to replace a subset of currently running (virtual) requests on some channels with some new coming requests that will have a larger total bid values, thus, we improve the revenue. When this is not feasible, we rely on a novel concept called $n$-weak preemption defined as follows to improve the value density of spectrum usage.

*Definition 2 ($n$-Weak Preemption):* Given a requests set $\mathcal{R}_1$, find a subset $\mathcal{R}'_1 \subset \mathcal{R}_1$ of requests $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\} \subseteq \mathcal{R}_1$ such that $\arg\max_{\{\mathbf{r}_1,\cdots,\mathbf{r}_i\}} \sum_{j=1}^{i} \frac{b_j}{t_j}$ and $n = \sum_{j=1}^{i} k_j$.

Let $\mathcal{R}_2$ be all requests that occupy the channels currently. Recall that we treated an original request that asked for $k_q$ channels as $k_q$ virtual requests, each of which asks for a single channel now with an adjusted bid $b'_q = b_q/k_q$. If there exists a subset $\mathcal{R}'_2$ of virtual requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\} \subseteq \mathcal{R}_2$ such that

$$\begin{cases} \sum_{j=1}^{i} \frac{b_j}{t_j} + \sum_{j=1}^{i'} (b'_j - \gamma(\mathbf{r}'_j)) > \Delta^{-\frac{1}{2}} \sum_{j=1}^{i'} b'_j \\ \sum_{j=1}^{i'} k'_j = n, \end{cases} \tag{2}$$

we preempt requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\}$ with requests $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\}$. Here $\gamma(\mathbf{r}'_j)$ is the penalty $\frac{t'_j}{t_j} b'_j$ to $\mathbf{r}'_j$ if virtual request $\mathbf{r}'_j$ is preempted at that time; $t'_j$ is the unfinished timeslots of this request. We say requests $\{\mathbf{r}_1, \cdots, \mathbf{r}_i\}$ *weakly preempt* requests $\{\mathbf{r}'_1, \cdots, \mathbf{r}'_{i'}\}$.

If $\Delta^{-\frac{1}{2}} \sum_{j=1}^{i'} b'_j - \sum_{j=1}^{i'} (b'_j - \gamma(\mathbf{r}'_j))$ is minimized in all feasible *$n$-weak preemptions*, we call it *minimum $n$-weak preemption*. Similarly, we can find the approximated *minimum $n$-weak preemption* in polynomial time: $\mathcal{R}'_1$ can be found approximately using a FPTAS where the adjusted "profit" of a request $\mathbf{r}_j \in \mathcal{R}_1$ is $b_j/t_j$, and $\mathcal{R}'_2$ can be found using a simple greedy in which we repeatedly free a channel assigned to a running request with the smallest value $(\Delta^{-\frac{1}{2}} - 1)b'_j + \gamma(\mathbf{r}'_j)$ till $n$ channels have been freed.

*Definition 3 (Critical Time):* For each request $\mathbf{r}_i = (b_i, k_i, a_i, t_i)$, the **critical time** of $\mathbf{r}_i$ is defined as $a_i + t_i \Delta^{-\frac{1}{2}}$.

According to the definition of *$n$-weak preemption*, after the *critical time* of $\mathbf{r}_i$, any coming request that asks for no more than $k_i$ channels could weakly preempt $\mathbf{r}_i$ if $\mathbf{r}_i$ is still running. Notice that we consider each running request that asks for multiple channels as multiple requests each of which asks for a single channel. For those multiple requests, their critical times are same as the critical time of the original request.

Algorithm 1 presents our online spectrum allocation method. Essentially, Algorithm 1 may do at most one *$n_1$-Strong Preemption* and at most one *$n_2$-Weak Preemption* where $n_1 + n_2 \leq m$. It always tries to do strong preemption first, then weak preemption.

### B. Performance Guarantees

We prove that the competitive ratio or our method is at least $\Omega(\Delta^{-\frac{1}{2}})$ in this subsection. To analyze the competitive ratio, we need to compute $\min_{\forall \mathcal{I}} \frac{\mathcal{P}(\mathcal{I})}{\mathrm{OPT}(\mathcal{I})}$. The main idea of our performance analysis is to prove the lower bound of $\mathcal{P}(\mathcal{I})$ (Lemma 4 and Lemma 5) and the upper bound of $\mathrm{OPT}(\mathcal{I})$.

To capture the relations between requests preemptions, we define a structure, called *request tree*. We omit the definition of request tree $\mathcal{T}(\mathbf{r})$ due to space limit.

Let $\mathcal{R}(\mathcal{I})$ be the set of all request tree roots for a given instance $\mathcal{I}$ of spectrum requests. Let $\mathcal{S}(\mathcal{I})$ be the set of all spectrum requests that are satisfied in instance $\mathcal{I}$ by our Algorithm 1.

*Lemma 4:* $\mathcal{P}(\mathcal{I}) \geq \frac{1}{2} \sum_{\mathbf{r} \in \mathcal{R}(\mathcal{I})} \mathcal{P}(\mathcal{T}(\mathbf{r}))$.

*Lemma 5:* $\sum_{\mathbf{r} \in \mathcal{R}(\mathcal{I})} \mathcal{P}(\mathcal{T}(\mathbf{r})) \geq \frac{1}{2}(1 - \frac{1}{c})\Delta^{-\frac{1}{2}} \sum_{\mathbf{r}_i \in \mathcal{S}(\mathcal{I})} b_i$

So we have $\mathcal{P}(\mathcal{I}) \geq \frac{1}{4}(1 - \frac{1}{c})\Delta^{-\frac{1}{2}} \sum_{\mathbf{r}_i \in S(I)} b_i$.

To analyze the competitive ratio, we also need to consider the profit made in optimal solution $\mathrm{OPT}(\mathcal{I})$. We divide it into two parts $\mathrm{OPT}_1(\mathcal{I})$ and $\mathrm{OPT}_2(\mathcal{I})$.

1) $\mathrm{OPT}_1(\mathcal{I})$ are requests which are granted by Algorithm 1 on instance $\mathcal{I}$. Obviously, $\mathrm{OPT}_1(\mathcal{I}) \leq \sum_{\mathbf{r}_i \in S(I)} b_i$.

**Algorithm 1** Efficient Online Spectrum Allocation Method $\mathcal{G}$

**Input**: A set of coming requests $\mathcal{R}_{in}$ arriving before time-slot $t$ and another set of requests $\mathcal{R}$ which occupy the channels currently.

**Output**: A set of requests $\mathcal{R}_{out}$ which occupy the channels at time-slot $t$.

```
R_out = ∅
for i = m to 0 do
    Perform minimum i-Strong Preemption by Definition 1.
    if we find a minimum i-Strong Preemption then
        Assume a set of requests R_1 ⊆ R_in preempts a set of
        requests R_2 ⊆ R in minimum i-strong preemption.
        R_in ⇐ R_in \ R_1 , R ⇐ R \ R_2 and R_out ⇐ R ∪ R_1.
        break;
for j = m − i to 1 do
    Perform minimum j-Weak Preemption by Definition 2.
    if we find a minimum j-Weak Preemption then
        Assume a set of requests R'_1 ⊆ R_in preempts a
        set of requests R'_2 ⊆ R in the minimum j-Weak
        preemption.
        R_out ⇐ R ∪ R'_1 \ R'_2.
        break;
Return R_out.
```

2) $\mathrm{OPT}_2(\mathcal{I})$ are requests which are not granted by Algorithm 1 on instance $\mathcal{I}$. We can prove that $\mathrm{OPT}_2(\mathcal{I}) \leq (2c+1) \sum_{\mathbf{r}_i \in S(I)} b_i$.

3) Therefore $\mathrm{OPT}(\mathcal{I}) \leq (2c+2) \sum_{\mathbf{r}_i \in S(I)} b_i$.

*Theorem 6:* The competitive ratio of Algorithm 1 is at least $\frac{c-1}{8c(c+1)}\Delta^{-\frac{1}{2}}$.y

*Theorem 7:* Let $c = \sqrt{2}$, the competitive ratio of Algorithm 1 is at least $\frac{6-4\sqrt{2}}{32}$ fraction of the optimum.

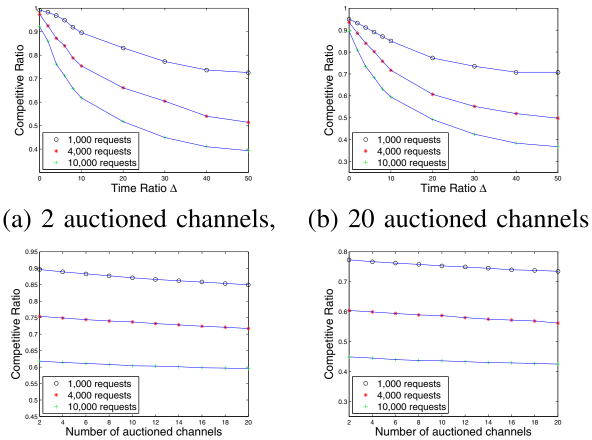
### C. Time Complexity of Our Method

The main component of our algorithm is to perform $i$-strong preemption and $j$-weak preemption, which involves solving knapsack problems. It is known that a FPTAS for knapsack problem [2] takes time $O(n^3/\epsilon)$, where $1 > \epsilon > 0$ is a small constant and $n$ is the number of items for the knapsack problem. Let $n_i(t)$ be the number of currently arriving requests, and all running requests at time $t$. Clearly, at time $t$, our algorithm takes time at most $O(m \cdot n_i(t)^3/\epsilon)$ where $m$ is the total number of channels. Then the overall time complexity of our method is $O(\Delta \cdot m \cdot n^3/\epsilon)$, because every running request will be considered in some knapsack problems for at most $\Delta$ timeslots. When $n_i(t)$ is at most a value $L$, then our method will take time at most $O(\Delta m n L^3/\epsilon)$.

## V. Other Models

### A. General Conflict Graph Model

Here we assume that the conflict-graph $H$ has bounded one-hop independence number $\alpha$. Performance upper bound still holds since it is proved by a special case. Algorithm 1 can be easily extended for this general model with asymptotically




(a) 2 auctioned channels, (b) 20 auctioned channels

(c) *Time ratio* $\Delta = 10$, (d) *Time ratio* $\Delta = 30$.

Fig. 1. The competitive ratios of our method in various cases.

same competitive ratio by redefining $n$-*Strong Preemption* and $n$-*Weak Preemption*. We omit the definitions due to space limit.

*Theorem 8:* Algorithm 1 is at least $\frac{c-1}{8\alpha c(c+1)}\Delta^{-\frac{1}{2}}$- competitive when interference graph $H$ has a one-hop independence number $\alpha$.

### *B. All-or-Nothing Preemption Model*

Our method can be also be easily extended to *all-or-nothing* preemption model with asymptotically same competitive ratio by redefining $n$-*Strong Preemption* and $n$-*Weak Preemption*. Details are omitted due to space limit.

*Theorem 9:* Algorithm 1 is at least $\frac{c-1}{16c(c+1)}\Delta^{-\frac{1}{2}}$- competitive in the second preemption model when conflict-graph $H$ is a completed graph.

## VI. Mechanism Design

For the our problem, each secondary user may lie on the bid, channel requirement, and time requirement. Our mechanism needs to ensure that each secondary user has incentives to declare his request truthfully. It was proved that a *monotone* allocation algorithm and a pricing scheme based on the *cut value* can ensure a truthful mechanism [14].

*Theorem 10:* Using the monotone FPTAS in [2] to solve the $n$-strong preemption and $n$-weak preemption, Algorithm 1 has the monotone property.

Our mechanism works as follows (1) we use Algorithm 1 as the spectrum allocation method, and (2) each admitted user $i$ is charged a payment equal to the cut value, *i.e.*, the minimum bid $\underline{b}_i$ for getting admitted. None admitted user will be charged 0. Preempted user will get compensation.

*Theorem 11:* In our mechanism, to maximize its profit, every user will not bid a price lower than its actual value.

## VII. Simulation Results

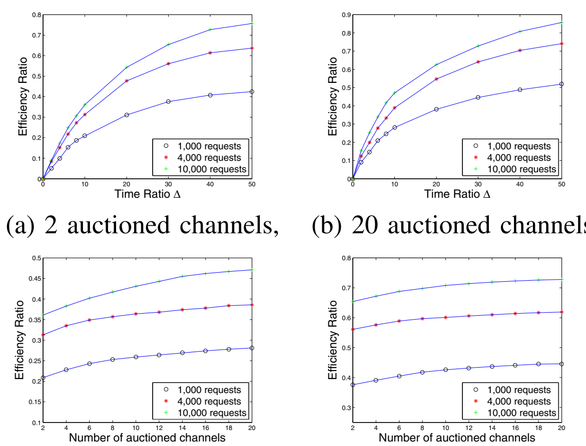
We conduct extensive simulations to evaluate the performance of our method in practice. Randomly deploy requests in a $5 \times 5$ square area. Two requests within distance 1 will interfere with each other. The bids are uniformly distributed in $[0, 1]$, time requirements are uniformly drawn from $[1, \Delta]$, channel requirements are uniformly drawn from $[1, m]$.y The total service time is always 200 time slots in our simulations.

### *A. Competitive Ratio of Our Method*

In Figure 1, we plot the competitive ratio for different $\Delta$ ((a)-(b)) and different number of channels ((c)-(d)). In most cases, our method can make a total profit that is more than 50% of the optimum. It implies that the performance of our method in simulations is much better than the theoretical bound.

Competitive ratio decreases significantly when $\Delta$ increases. This coincides with our theoretical results. The competitive ratio decreases when there are more requests because our method is conservative and only tries to maximize the minimum. With more requests, the optimal solution makes more profit, but our method just tries to satisfy the bound.

### *B. Efficiency Ratio of Our Method*




(a) 2 auctioned channels, (b) 20 auctioned channels

(c) *Time ratio* $\Delta = 10$, (d) *Time ratio* $\Delta = 30$.

Fig. 2. The efficiency ratio of our method in various cases.

In Figure 2, we plot the efficiency ratios for different y$\Delta$ ((a)-(b)) and different number of channels ((c)-(d)). The efficiency ratio increases significantly when *time ratio* $\Delta$ increases. This is because larger $\Delta$ results in more conflicts in time requirement, and more conflicts imply easier replacements. Then the payment based on cut value will be better and efficiency ratio increases.

Similarly, the efficiency ratio also increases when the number of auctioned channels increases. When users ask for more channels, more conflicts appear in the process of auction. Thus, it is easier to find a replacement and the efficiency ratio increases. Moreover, more requests also result in more conflicts. Thus the efficiency ratio increases when more requests take part in the auction.

## VIII. Literature Reviews

Yuan *et al.* [26] introduced the concept of a time-spectrum block to model spectrum allocation problem. They presented both centralized and distributed protocols for spectrum allocation and show that these protocols are close to optimal in most scenarios. Li *et al.* [16] designed PTAS or efficient approximation algorithms for different versions of dynamic spectrum assignment problems. They also showed how to design truthful mechanism based on those algorithms. Zhou *et al.* [27] propose a truthful and efficient dynamic spectrum auction system to serve many small players. In [28], Zhou and Zheng designed truthful double spectrum auctions where multiple parties can trade spectrum based on their individual needs. All these results are based on offline models.

Our studies are also related with online job scheduling problems which still receive a lot of research interest. Various online scheduling problems (*e.g.*, [1], [6], [8], [10], [13], [15], [25]) focus on optimizing different objective functions, *e.g.*, *makespan*, which is the length of the schedule. Another model aims to maximize the profit or number of completed jobs as our model. There are different variants: preemption-restart, preemption-resume, and preemption-discard. Woeginger [20] studied an online model of maximizing the profit of finished jobs where there is some relationship between the weight and length of job. He provided a $4$-competitive algorithm for tight deadline case, and gave a matching lower bound. Hoogeveen *et al.* [9] gave a $\frac{1}{2}$-competitive algorithm which maximizes the number of early jobs. They assume that preemption is allowed while no penalties will be charged. Chrobak *et al.* [3] gave a $\frac{2}{3}$-competitive algorithm which maximizes the number of satisfied jobs with uniform length in the *preemption-restart* model.

The most similar works are [22] and [23]. However, in these works, only single channel case is studied, which is a special case of our model.

## IX. Conclusions

This work studied the online spectrum scheduling in multiple-channels wireless networks. When the primary users reserved some channels for some time-slots, our methods also work correctly: all requests that conflict with these reservations will be discarded. This work is just a stepstone for a number of interesting questions. An interesting question is to design online algorithm whose performance is asymptotically optimum for a general penalty function. Another question is to study the case where each channel is unique. This case is similar to weighted set packing problems. The last but the not the least challenge is to design efficient online spectrum allocation and auction protocols with statistical performance assurance when we know the probability distribution of the bids (the bid value, the asked time-slots, and the arrival times).

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