TODA: Truthful Online Double Auction for Spectrum Allocation in Wireless Networks

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Abstract—The spectrum usage by a secondary user often happens in a certain geographical region and in a certain time interval, and the requests often come in an online fashion. Considering the selfish behaviors of primary users and secondary users, it is imperative to design *online* double spectrum auction methods. The most significant challenge is how to make the online double auction economic-robust (truthful in particular). Unfortunately, existing designs either do not consider the online requests or become untruthful when applied to scenarios when both primary users and secondary users could be selfish.

In this paper, we address this problem by proposing TODA, a general framework for truthful online double auction for spectrum allocation. We assume that there is a central auctioneer, and the arrivals of secondary users' requests follow Poisson distribution. Upon receiving online spectrum requests, the central auctioneer will decide immediately which secondary and primary users will win the auction, and match winning primary users and secondary users, as well as decide how much secondary users should pay and primary users should get. To preempt existing spectrum usage is *not* allowed. We study the case in which the conflict graph of secondary users is a complete graph, which occurs in the urban area where the distribution of the secondary users is very dense. In this case, we design strategyproof (truthful) mechanisms for both the primary users and secondary users. To the best of our knowledge, we are the *first* to design truthful online double auction mechanisms for spectrum allocation. Our simulation results show that the expected social efficiency ratio of our mechanism is always above 80% compared with the off-line VCG mechanism and the spectrum utilization ratio is around 70% when the system is highly loaded.

Index Terms—Wireless networks, spectrum, online allocation, double auction.

I. INTRODUCTION

The frequency spectrum has been chronically regulated with static allocation policies since the early $20th$ century. In the past decade, the FCC (Federal Communications Commission) and its counterparts across the world have been using *single-sided* auctions to assign spectrum to wireless service providers in terms of *predetermined* national/regional long-term leases. With the recent fast growing spectrum-based services and devices, the remaining spectrum available for future wireless services is being exhausted. However, the current fixed spectrum allocation scheme leads to significant spectrum *white spaces* where many allocated spectrum blocks are used only in certain geographical areas and/or in brief periods of time. Recognizing that the traditional spectrum management process can stifle innovation, FCC has proposed new spectrum management models [19].

Spectrum subleasing is widely regarded as a potential way to share spectrum. In this paper, we show that auctions can be used when some primary user has white space in some specific area or during some specific time period. By auctions, spectrum can be dynamically redistributed among multiple parties to meet their own demands. In our model, a central auctioneer runs *double* spectrum auctions to enable multiple primary users (sellers) and secondary users (buyers) trade spectrum dynamically. By multiplexing spectrum supply and demand in time and space, dynamic auctions can significantly improve spectrum utilization.

Previous studies on spectrum assignment (*e.g.* [11], [14], [22], [25]) only considered single-sided spectrum auctions, which assume that primary users will always trust the central authority and be satisfied with the outcome of the auction. However, in practice, primary users are usually *selfish* and want to participate in the auction. So it is reasonable to design double auctions for spectrum allocation. Unfortunately, previous truthful double auction designs [2], [15], [7], [26] only consider the single-round case, which is not fit for the continuous spectrum auction where secondary users come in a stochastic way and in an on-line fashion. There are also some papers focused on online double auctions [5], but they only focus on single item. In the case of spectrum allocation, we must consider requested time durations of secondary users, so this previous work can not be directly used in double auctions for spectrum allocation. Blum *et al.* [4] present online matching algorithms whose competitive ratios depend on the spread in offer valuations. This earlier work does not consider incentive-compatibility, and is presented purely from an algorithmic perspective.

In auctions, truthfulness (or strategyproofness) is one of the most critical properties. An auction without this property is extremely vulnerable to market manipulation and produces very poor outcomes, shown by both economic theory and concrete examples [12]. Due to the fact that spectrum is fundamentally different from conventional goods as stated before, designing economic-robust double auctions for spectrum allocation is more challenging.

In this paper, we propose a mechanism $\mathcal{M}$ for online double spectrum auction which achieves *truthfulness, ex-post budget balance*, and *individual rationality*. We make the following assumptions about spectrum subleasing: (1) spectrum allocation is non-preemptive, (2) time is slotted and only integer number of time slots will be requested, (3) the arrivals of spectrum requests of secondary users follow Poisson distribution, and all primary users join at the beginning of the auction and leave until the end of the auction, (4) all decisions cannot be revoked (thus a rejected request cannot be reconsidered later), (5) the probability distributions of biding and time requirement can be learned by the central auctioneer based on previous knowledge [16]. For that preemption is not allowed, the central auctioneer cannot terminate current running request(s) to satisfy a new coming request. To the best of our knowledge, we are the *first* to design truthful online double auction mechanisms for spectrum allocation.

The main contributions of this paper are as follows. We study the case that the conflict graph of secondary users is a complete graph. In this model, for any spectrum channel, if we have allocated it to some secondary user u_i, then before u_i finished using the channel, we can not allocate the spectrum channel to any other secondary user u_j. Here the conflict graph is $H = (\mathcal{V}, E)$, where $\mathcal{V}$ is the vertex set corresponding to the requests of secondary users and $(v_i, v_j) \in E$ if and only if the requests of v_i and v_j conflict with each other. For that the requests of the spectrum channels in our model not only contain the bidding price for one time-slot but also contain the time duration of using the spectrum, the classical truthful double auctions, such as McAfee double auctions, cannot be directly used in our model. However, we can borrow the idea of McAfee double auctions. Our idea is that we consider the expected revenue of each primary user rather than the exact revenue by calculating the expected *earning* of every primary user at the time when some requests come. Here earning is defined as the money a primary user collects, which does not consider the cost. After calculated the expected earnings of all primary users, we construct a matching bipartite graph (Section III) based on them, which represents the potential winning secondary users (candidates) for each primary user. Based on this matching bipartite graph, we admit secondary users and primary users and decide how much should the winner pay. Our mechanism achieves all the desiderata: strategyproofness, ex-post budget balance, and individual rationality. We then perform simulations to show our mechanism is efficient in both social efficiency ratio (defined in Section II) compared with the optimal offline VCG meethod, and spectrum utilization ratio.

The rest of the paper is organized as follows. In Section II, we define the network models and the spectrum allocation problems to be studied. In Section III, we present TODA for online double spectrum auction for spectrum allocation, and we analysis our method in Section IV. Our simulation studies are reported in Section V. We review the related work in Section VI and conclude the paper in Section VII.

II. PRELIMINARIES

In this section, we discuss the network model for online double auction and the bidding model of primary and secondary users, and formulate our problem.

A. Wireless Network Model

Consider a wireless network consisting of some primary users $U = \{u_1, u_2, \cdots, u_m\}$, each of whom holds the usage right of one spectrum channel and is willing to sublease the usage of the channel to secondary users for time interval $[0, T]$. The channels are identical. Here the time is slotted (discrete), *i.e.* the time interval $[0, T] = \{0, 1, 2, \ldots, T\}$. The wireless network also consists of some secondary users $V = \{v_1, v_2, \cdots, v_n\}$ who want to lease the right of using some channel for some period of time in some region. Note that different from previous work, in our network model, secondary users will come in an online fashion. In practice, each primary user u_i is selfish and has a *true valuation* $\tilde{s}_i$ for the usage of its spectrum per time-slot. Thus the single-sided auction which only cares the selfish behavior of secondary users will be no longer proper under this model. Here we design double spectrum auction mechanisms in which buyers enter competitive bidders and sellers enter competitive offerers simultaneously, and there is a central authority who performs the double spectrum auction.

We assume that the arrivals of requests proposed by secondary users are independent and identically distributed (*i.i.d.*) and follow *Poisson* distribution with the expected arrival rate λ. The bidding price and the requested time slots of secondary users also follow some specific probability distributions. We assume the central auctioneer knows all the probability distributions (, otherwise it can learn them based on the previous knowledge using the method proposed by [16]). Primary users join in the auction at the beginning, submit their asking prices to the central auctioneer before auction starts, and remain in the auction until the end.

Since the conflict among primary users has been solved by the auction performed by FCC, in our work, we assume the conflict of spectrum usage only happens among the secondary users, called secondary conflict. The secondary conflict depends on the locations and requested time periods of the secondary users. For example, under protocol interference model [9], if secondary user v_i is in the interference range of secondary user v_j, they can not use a same channel simultaneously. Under the physical interference model, some concurrent usages of one channel interfere with one another if and only if the Signal-to-Interference-plus-Noise-Ratio (SINR) at the receivers is below some acceptable threshold. In this paper, we assume that the location of each secondary user is *fixed*. This location-dependent conflict will be modeled by a conflict graph $H = (\mathcal{V}, E)$, where $\mathcal{V}$ is the vertex set corresponding to the requests of secondary users, and two nodes v_i and v_j form an edge $(v_i, v_j) \in E$ if and only if they conflict with each other, *i.e.*, they cannot use a same channel simultaneously.

B. Online Spectrum Bidding Model

We assume there are m primary users in the auction, and one primary user contributes one distinct channel; thus there are m channels in the network. Each primary user u_j has a true valuation $\tilde{s}_j$ of his spectrum channel usage per time slot. Primary user u_j bids asking price s_j for selling per time-slot of the spectrum usage to the auctioneer. Since primary user u_j is selfish, asking price s_j is not necessarily equal to the true valuation $\tilde{s}_j$ if cheating could improve its profit. In this paper, we assume that the asking prices of all primary users are fixed during the auction. We leave the problem model of adaptive asking prices of primary users as a future work.

In our model, each secondary user only requests one channel. Let $\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_i, \cdots$, be the sequence of all requests of secondary users. Each request $\mathbf{r}_i = (v_i, a_i, b_i, t_i)$ is claimed by a secondary user v_i at the time a_i, who bids b_i for the *per time unit* usage of the channel and requests to use t_i time-slots. Note that the time is discrete. The total money secondary user v_i willing to pay is $b_i \cdot t_i$ for spectrum channel usage. After received a request from some secondary user, the auctioneer needs to give the result, whether it is admitted or not, immediately. And the requests cannot be recalled, *i.e.*, if the request is rejected, then it will never be considered again. For most of our discussions we will omit v_i in $\mathbf{r}_i$ when it is clear from the context, or not needed in the notation. So the request of secondary user v_i is simplified as $\mathbf{r}_i = (a_i, b_i, t_i)$.

Each secondary user v_i has its true valuation $\tilde{b}_i$ for *per time unit* usage of a spectrum channel, and bid b_i for using a unit time. Here b_i is not necessarily equal to $\tilde{b}_i$. In our model, secondary users can also be untruthful on its required time slots, but could only bid a longer time period. The reason that we assume a secondary user cannot announce a shorter requested time duration is that, even if it wins in the auction by requesting a shorter time duration, the secondary cannot finish his own work; thus he will always get negative utility. For that the spectrum channels are assumed identical, the central auctioneer can assign any available channel to a winning secondary user. Note that this assumption is valid although the asking prices of different primary users could be different (the difference may be resulted from the different costs of primary users when buying the spectrum channels). The auction is sealed-bid and private. Bidders submit their bids privately to the auctioneer without any knowledge of others and do not collude.

C. Problem Formulation

In this paper, we will study the complete conflict graph model for secondary users; we leave the general conflict graph model as a future work. The objective of this paper is to design an online double spectrum auction satisfying the economic properties of *truthfulness, individual rationality, and budget balance*, which are three critical properties required for economic-robust double auction [2] [12]. The three economic properties are defined as following:

(1) **Truthfulness:** A double auction is *truthful* if no matter how other players bid, **no** seller u_i or buyer v_j can improve its own profit (utility) by bidding untruthfully ($s_i \neq \tilde{s}_i, b_j \neq \tilde{b}_j$, or bid a untruthful requested number of time-slots). Truthfulness is also called strategyproofness. In our problem, truthfulness requires that: (1) the secondary users report their true required number of time slots (called time-SP), (2) primary and secondary users report their true valuation for the per time unit usage of spectrum channel (called value-SP).

Truthfulness is essential to resist market manipulation and ensure auction fairness and efficiency. In untruthful auctions, selfish bidders can manipulate their bids to obtain outcomes that favor themselves but hurt others. In truthful auctions, the dominate strategy for bidders is to bid truthfully, thereby eliminating the fear of market manipulation and the overhead of strategizing over others. With the true valuations, the auctioneer can allocate spectrum efficiently to buyers who value it the most.

(2) **Individual Rationality:** A double spectrum auction is *individual rational* if no winning seller is paid less than its bid (per time-slot) and no winning buyer pays more than its bid (per time-slot), *i.e.* the utility of the sellers and buyers is no less than 0. If we let $\mathbf{p}_j^s > 0$ to denote the total payment of secondary user v_j, and t is the total time requested, then individual rational requires $\mathbf{p}_j^s - b_j \cdot t < 0$. In our probability model, the individual rationality of a primary user means that the expected profit is no less than 0. Here the expected profit is defined as the expected earning subtract the cost of the spectrum channel $\tilde{s} \cdot T$. This property guarantees non-negative utilities for bidders, providing them incentives to participate.

(3) **Ex-post Budget Balance:** A double spectrum auction is *ex-post budget balanced* if the auctioneer's profit $\phi \geq 0$. The profit ϕ is defined as the difference between the revenue collected from buyers and the expense paid to sellers, *i.e.* $\phi = \sum_{i=1}^{m} \mathbf{p}_i^p + \sum_{j=1}^{n} \mathbf{p}_j^s \geq 0$. Observe that here $\mathbf{p}_i^p < 0$, denoting that the auctioneer has to pay $-\mathbf{p}_i^p$ to the primary user u_i; $\mathbf{p}_j^s > 0$, denoting that auctioneer collects $\mathbf{p}_j^s$ from secondary user v_j.

This property ensures that the auctioneer has incentives to set up the auction. Note that in practice the auctioneer can charge a transaction fee to (winning) bidders. For simplicity, we do not include this charge in the profit computation.

Instead of the worst case performance, we focus on the expected performances, *i.e.* the total expected earning of the primary users. Based on the expected earning, we design an online double auction which guarantees to achieve these three economic properties under our complete conflict graph model.

To evaluate the performance of an online mechanism, we compare it with *offline VCG* mechanism (Vick) which will maximize the total social efficiency (but not necessarily strategyproof in the online model) in our simulation section. The performance of an online mechanism $\mathcal{M}$ is measured by its *social efficiency* $\mathrm{Eff}(\mathcal{M})$, defined as the total true valuations of all winners, *i.e.* $\mathrm{Eff}(\mathcal{M}) = \sum_i x_i b_i$. The *social efficiency ratio* of a mechanism $\mathcal{M}$ is $\mathrm{Eff}(\mathcal{M}, \mathrm{Vick}) = \mathrm{Eff}(\mathcal{M}) / \mathrm{Eff}(\mathrm{Vick})$. In our simulation part, we will study the expected total social efficiency ratio of our mechanism $\mathcal{M}$.

III. OUR AUCTION MECHANISM $\mathcal{M}$

The complete conflict graph model requires that if we admit the request of one secondary user on some spectrum channel for some period of time, we cannot admit any other request on the same spectrum channel in that time duration. We propose our mechanism $\mathcal{M}$ in this section.

Recall that we assume at the very beginning of the auction, each primary user u_i submits their asking price s_i to the central auctioneer which does not change during the auction. After the auction started, secondary users come in a stochastic manner; the arrivals of them follow Poisson distribution with arrival rate λ. The bidding price and requested time intervals follow some specific distribution which is known (or can be learned [16]) by the central auctioneer based on previous knowledge. For example, it is possible that, based on historical data, the bidding price of the incoming requests satisfies the Normal distribution with mean value μ and standard deviation σ, and the requested number of timeslots satisfies the Uniform distribution ranging from t_{min} to t_{max}.

At the beginning of the auction, all the primary users submit their asking price $s_1, s_2, \ldots, s_m$ to the central auctioneer. The first step of our mechanism $\mathcal{M}$ is that, for each primary user u_i, the central auctioneer calculates the expected earning $V_i(t), \forall 0 \leq t \leq T$, for leasing the spectrum channel from time t to the end of auction T. In other words, at the beginning of the auction, the central auctioneer prepares an $m \times T$ matrix, and in each cell (i, t) is the expected earning $V_i(t)$ based on the historical data. Note $V_i(t)$ denotes the expected earning from time t to T of primary user u_i. The calculation is by *Dynamic Programming (DP)* as follows. Let X_k denote the event that there are k requests arrived at the beginning of time slot t, and Y_t denote the event that the requiring time slots is t, and Z_b denote the event that the bidding value of a request is b. Then the expected earning of primary user i is as follows:

$$V_i(t) = \sum_{k=0}^{+\infty} \mathbf{Pr}\,(X_k)(\sum_{t_n=1}^{T-t} \mathbf{Pr}\,(Y_{t_n})(b_i(k) \cdot t_n + V_i(t+t_n))).$$
(1)

Here $b_i(k) \geq s_i$ is an expected bidding value based on the arrival rate of the requests and the bidding distribution of the secondary users. We use $b_i^m(k)$ to denote the expected highest bidding value when the bidding prices of the k requests are all above the asking price of primary user u_i. Clearly, $b_i^m(k)$ can be calculated as below:

$$\int_{s_i}^{b_i^m(k)} \mathbf{Pr}\,(Z_x)dx = \frac{k}{k+1} \cdot \int_{s_i}^{+\infty} \mathbf{Pr}\,(Z_x)dx.$$
(2)

Note that the value $b_i^m(k)$ is the ideal case that will not always be achieved (because sometimes not all the bidding prices will be above the asking price s_i of primary user u_i). We introduce a parameter $0 < \delta \leq 1$. Then $b_i(k)$ is computed by

$$b_i(k) = \max\{\delta \cdot b_i^m(k), s_i\}.$$

After $V_i(t)$ and $b_i(k)$ are given, Lemma 1 follows directly:

Lemma 1: $V_i(t)$ is monotonically decreasing as t grows, and $b_i(k)$ is monotonically increasing as s_i grows.

Based on Lemma 1, we get the following corollary:

Corollary 2: If $s_i \leq s_j$, then for any time t, $V_i(t) \leq V_j(t)$.

Proof: We prove by induction on t. First when $t = T$, $\forall i, V_i(T) = 0$, the corollary holds. Assume that $\forall t, t > t^0$, the corollary holds, and $s_i \leq s_j$, then for the time t^0 we get

$$
\begin{aligned}
V_i(t^0) &= \sum_{k=0}^{+\infty} \mathbf{Pr}\,(X_k)(\sum_{t_n=1}^{T-t} \mathbf{Pr}\,(Y_{t_n})(b_i(k) \cdot t_n + V_i(t^0 + t_n))) \\
&\leq \sum_{k=0}^{+\infty} \mathbf{Pr}\,(X_k)(\sum_{t_n=1}^{T-t} \mathbf{Pr}\,(Y_{t_n})(b_j(k) \cdot t_n + V_j(t^0 + t_n))) \\
&= V_j(t^0)
\end{aligned}
$$

So the corollary holds for every $t = 1, \ldots, T$. ∎

After the auctioneer prepared $V_i(t), \forall 1 \leq i \leq m, 0 \leq t \leq T$, it will start the auction. The secondary users come in a stochastic way. At the beginning of each time slot, if some secondary users came, a *decision phase* happens. In other words after the auction begins a series of decision phases will happen. The admission method of our mechanism $\mathcal{M}$ for each decision phase contains two steps: (1) the *matching bipartite graph $BG_t(B, S, E)$ construction* step, (2) admission and charge decision making step, all of which will be discussed in below.

A. Matching Bipartite Graph Construction

When a secondary user v_j comes, it submits its request to the central auctioneer. Then the auctioneer computes $V_i^j(t) = b_j \cdot t_j + V_i(t + t_j)$ which is the expected earning after the primary user u_i admits the secondary user v_j. The auctioneer will keep m values $V_i^j(t)(i = 1 \ldots m)$ for each bidder v_j who comes at the decision phase starting at time t. If $V_i^j(t)$ is greater than the expected earning $V_i(t)$, then it is better for primary u_i to admit the secondary user v_j than to wait next request. Under this condition, we call v_j is a candidate to u_i.

Definition 1: If the expected earning for the primary user u_i to admit bidder v_j is no less than the expected earning at the current time t, *i.e.* $V_i^j(t) \geq V_i(t)$, then bidder v_j is a *candidate* (potential winner) to primary user u_i.

Here we use a bipartite graph to represent the relationship of "is-a-candidate-to", and we call this bipartite graph the *matching bipartite graph $BG_t(B, S, E)$*. In matching bipartite graph $BG_t(B, S, E)$, there are two sets of vertices B and S. The vertex set $B = \{\beta_1, \beta_2, \ldots, \beta_n\}$ denotes the set of n buyers arrived at the beginning of time slot t, and the vertex set $S = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_k\}, k \leq m$, denotes the set of k primary users whose channels are available (not allocated) at the beginning of time slot t. If buyer v_j is a candidate to primary user u_i, then we add an edge (β_j, ς_i) to the edge set E. A key property of $BG_t(B, S, E)$ is shown in Lemma 3.

Lemma 3: For two sellers u_i and u_j, if $s_i < s_j$, then for any candidate v_k to primary user u_j is also a candidate to primary user u_i.

Proof: For any request v_k is a candidate to primary user j, it must have $V_j^k(t) = b_k \cdot t_k + V_j(t + t_k) > V_j(t)$. $V_i^k(t) - V_i(t) = b_k \cdot t_k + V_i(t + t_k) - V_i(t) = b_k \cdot t_k - V_i(t, t + t_k - 1) > b_k \cdot t_k - V_j(t, t + t_k - 1) > 0$. Here $V_i(t_1, t_2)$ denotes the expected earning of primary user u_i from time t_1 to t_2. So the lemma holds. ∎

The following Corollary 4 can be directly got from Lemma 3.

Corollary 4: For any two elements $\varsigma_i, \varsigma_j \in S$, the corresponding asking prices satisfy $s_i < s_j$, $\forall \beta_k \in B$, if $(\beta_k, \varsigma_j) \in E$, then $(\beta_k, \varsigma_i) \in E$.

B. Making Admission and Charging Decision

After $BG_t(B, S, E)$ is constructed, we are ready for making the admission and charging decisions. In this section, we give the admission method and the charging method.

One trivial idea is to find the *maximum weighted matching* in $BG_t(B, S, E)$ and admit the matched (primary user,secondary user) pairs. However, we found that trivially doing the maximum matching will not give us a *strategyproof* mechanism, for that the secondary user could increase its utility by decreasing its bidding price to match a primary user with a lower asking price. Here we borrow the idea of McAfee double auction to design our mechanism which satisfies the three desiderata: truthfulness, individual rationality, and ex-post budget balance.

In our mechanism $\mathcal{M}$, we first sort the set of S in increasing order based on the asking price $s_i, \forall i \in \{1, 2, \cdots, k\}$. Let $|\beta_i|$ denote the number of edges incident with vertex $\beta_i \in B$ in the matching bipartite graph $BG_t(B, S, E)$. We then sort the set of B in decreasing order based on $|\beta_i|$. We re-number the elements of S and B after this sorting, *i.e.* $\forall \varsigma_i, \varsigma_j \in S, 1 \le i \le j \le k$, then $s_i \le s_j$ and $\forall \beta_{i'}, \beta_{j'} \in B, 1 \le i' \le j' \le n$, then $|\beta_{i'}| \ge |\beta_{j'}|$.

From now on, the matching bipartite graph $BG_t(B, S, E)$ is referred to the sorted bipartite graph. By Lemma 3, we know that if $(\beta_j, \varsigma_i) \in E$ then $\forall l \le i, (\beta_j, \varsigma_l) \in E$, so the Bipartite Graph $BG_t(B, S, E)$ must be in the form similar as Fig. 1.
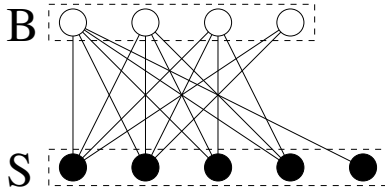

Fig. 1. The possible bipartite graph after vertex sorting.

In the bipartite graph $BG_t(B, S, E)$, we find that if E is not empty ($E \ne \Phi$), there must be some edge(s) with the form as (β_i, ς_i). We call such edges as the *diagonal pairs* (because that if we use a adjacent matrix to denote $BG_t(B, S, E)$ then all such edges (the "1"s) lie in the diagonal of the matrix). Let N be the number of the diagonal pairs in $BG_t(B, S, E)$. For example, in Fig. 1 $N = 3$. We call N the *matching number*.

Let $BG_t^N(B^N, S^N, E^N) \subseteq BG_t(B, S, E)$ to denote the bipartite graph induced by $B^N = \{\beta_1, \beta_2, \ldots, \beta_N\}$ and $S^N = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_N\}$. It is trivial that $BG_t^N(B^N, S^N, E^N)$ is a complete bipartite graph $K_{N,N}$. For each buyer v_i corresponding to $\beta_i \in B^N$, we calculate the expected earning $V_N^i(t)$ for seller u_N corresponding to $\varsigma_N \in S^N$. Recall that $V_N^i(t)$ is the expected earning that seller u_N admits buyer v_i. We next perform a second sorting on the N elements of vertex set B^N

based on $V_N^i(t)$ in decreasing order. We use $\{\beta_1', \beta_2', \ldots, \beta_N'\}$ to denote the order after the second sorting. Since that the bipartite graph $BG_t^N(B^N, S^N, E^N)$ is a complete bipartite graph $K_{N,N}$, it is trivial to get the matching number is also N, *i.e.* $(\beta_i', \varsigma_i) \in E, \forall 1 \le i \le N$.

From now on, again, we use the index of the newly sorted bipartite graph $BG_t^N(B^N, S^N, E^N)$, *i.e.* $\forall \beta_i \in B$ is the ith elements of the newly sorted vertex set B^N. Similar to the McAfee double auction, the central auctioneer admits the first $(N - 1)$ pairs of bidders (primary and secondary users) and charges the admitted pairs based on the Nth pair (but not exactly the bids of the Nth pair). The charging method is as follows.

We use $\mathbf{p}_j^s$ to denote the payment of secondary user v_j, and $\mathbf{p}_i^p$ to denote the payment of primary user u_i. Note that $\mathbf{p}_i^p < 0$ represents the value paid to primary user u_i for subleasing the spectrum channel to secondary user v_i. If the requested time-slots is t_j, the admitted secondary user v_j will be charged

$$\mathbf{p}_j^s = b_N \cdot t_N + V_N(t + t_N) - V_N(t + t_j). \qquad (3)$$

If the requested time-slot of the admitted secondary user v_i is t_i, the admitted primary user i will be paid

$$\mathbf{p}_i^p = -V_N(t) + V_N(t + t_i). \qquad (4)$$

And the non-admitted primary (secondary) users will be charged 0. The whole algorithm of our mechanism $\mathcal{M}$ is concluded in Algorithm 1.

IV. THEORETICAL ANALYSIS OF MECHANISM $\mathcal{M}$

In this section, we prove that our mechanism $\mathcal{M}$ satisfies all the desiderata.

First, we prove the most important economic property strategyproofness of our mechanism $\mathcal{M}_i$ which requires both *time*-strategyproofness and *value*-strategyproofness. To prove our mechanism $\mathcal{M}$'s truthfulness, we need to show that for any secondary user v_i or primary user u_j, it cannot improve its utility by bidding other than its true valuation. In other words, the agent's dominant strategy is to report its true bid value (called *value-SP*) and true time requirement (called *time-SP*). For this, we first show that its winner determination is monotonic for both sellers and buyers and the pricing is bid-independent. Using these two claims, we then prove the truthfulness.

The following two lemmas summarize the monotonicity of our mechanism $\mathcal{M}$'s winner determination.

Lemma 5: Given the bids set $\{b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n\}$ and the set of asking prices $\{s_1, s_2, \ldots, s_k\}$, if buyer v_i wins the auction by bidding b_i, then it will win by bidding $b' > b_i$.

Proof: For that we only admit the first $N - 1$ pairs of the vertexes of the sorted bipartite graph, so whether admit or not is merely depended by the position in the sorted vertexes set. If the secondary user v_i bids higher than before, then there will be no less edges incident with the corresponding vertex in the set of B, so its position index in B will not increase, *i.e.* we will also admit it. So the lemma holds. ∎

Similarly, we can prove the corresponding lemma for the buyer side.

Algorithm 1 Spectrum Allocation Mechanism $\mathcal{M}$ for Double Spectrum Auction in Complete Conflict Graph.

Input: The asking prices of the primary users and the probability distributions required.

Output: The spectrum channel allocation method and charging mechanism.

1: Primary users submit their asking prices $\{s_1, \ldots, s_m\}$ to the central auctioneer.
2: For each primary user u_i, the central auctioneer calculate $V_i(t), \forall t, 0 \leq t \leq T$.
3: **while** $t \neq T$ **do**
4:   **if** A decision phase starts **then**
5:     For each request of secondary user v_j, central auctioneer calculates the expected earning $V_i^j(t)$ after admitted v_j for each primary user u_i.
6:     Use Bipartite Graph $BG_t(B, S, E)$ to represent the relationship of "is-a-candidate-to".
7:     Sort the set of S in increasing order based on the asking price $\{s_1, s_2, \ldots, s_k\}$, here $k \leq m$ is the number of available spectrum channels.
8:     Sort the set of B in decreasing order based on the number of edges($|b_j|$) incident with b_j.
9:     After first sorting the central auctioneer got the matching number N, then begins a second sorting based on $V_N^i(t), \forall 1 \leq i \leq N$.
10:     Admit the first $N-1$ pairs and allocate the spectrum corresponding to s_i to the buyer corresponding to b_i.
11:     Each admitted primary user i is charged by $\mathbf{p}_i^p = -V_N(t) + V_N(t + t_i) < 0$, and each admitted secondary user j is charged by $\mathbf{p}_j^s = b_N \cdot t_N + V_N(t + t_N) - V_N(t + t_j) > 0$.
12:     The other bidders (primary and secondary users) is charged 0.

Lemma 6: Given the set of bids $\{b_1, b_2, \ldots, b_n\}$ and the set of asking prices $\{s_1, s_2, \ldots, s_{j-1}, s_{j+1}, \ldots, s_k\}$, if seller u_j wins the auction by asking s_j, then it will win by asking $s' < s_j$.

The following two lemmas summarize the pricing is bid-independent for winning buyers and sellers.

Lemma 7: Given the bids set $\{b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n\}$ and $\{s_1, s_2, \ldots, s_k\}$, if buyer v_i wins the auction by bidding b_i and b', then the mechanism $\mathcal{M}$ will charge the same for both.

Proof: Similar as the proof of Lemma 5, by formula (3), the charging method of $\mathcal{M}$ is only depend on the Nth buyer's bid b_N and the spectrum channel N. So whether we change the bid value of buyer v_i or not, only if it will be admitted, the charging price will be the same. ∎

Lemma 8: Given the set of bids $\{b_1, b_2, \ldots, b_n\}$ and $\{s_1, \ldots, s_{j-1}, s_{j+1}, \ldots, s_k\}$, if seller u_j wins the auction by asking s_j and s', then our mechanism $\mathcal{M}$ will charge the same for both.

The proof of Lemma 8 is similar as that of Lemma 7.

Based on the above lemmas, we now prove the main results on our mechanism $\mathcal{M}$'s truthfulness. We use *quasi-linear*

utility functions; the expected utility function of primary user u_i is $\mathcal{U}_i^p = V_i(0) - \tilde{s}_i \cdot T$, and the utility function of a winning secondary user v_j is $\mathcal{U}_j^s = b_i \cdot t_i - \mathbf{p}_i^s$ and for a loser the utility function is 0.

In the following proofs of the lemmas, we will use $\tilde{b}_i$ (resp. $\tilde{s}_i$) to denote the true valuation of the secondary user v_i (resp. primary user u_i), and use b_i' (resp. s_i') to denote the untruthful valuation of secondary user v_i (*resp.* primary user u_i). Similarly, we use $\mathcal{U}'$ to denote the untruthful utility. In the following lemmas, we will prove our mechanism $\mathcal{M}$ is truthful.

We first prove that our mechanism $\mathcal{M}$ is truthful for primary users. Please note that the sellers can only be untruthful on its asking price in our double auction.

Lemma 9: Mechanism $\mathcal{M}$ is truthful (value-SP) for primary users (sellers).

Proof: To prove the lemma holds, it is equivalent to prove that on every case listed above the utility function $\mathcal{U}_i$ when seller u_i is truthful will be no less than that when it lies on its asking price s_i.

Get admitted when truthful: In this case, the seller will be admitted being truthful. If it get admitted when bidding untruthfully, by Lemma 8 we know that the charging policy for seller u_i of $\mathcal{M}$ is independent with the asking price s_i. So no matter how seller u_i lies on its asking price, the utility function $\mathcal{U}_i^p$ will not change. If it cannot be admitted when bidding untruthfully, by the Theorem 14, we know that the utility function $\mathcal{U}_i^p \geq 0 = \mathcal{U}_i'^p$. So our mechanism is truthful in this case.

Not admitted when truthful: In this case, the seller cannot be admitted if bids truthfully. We first consider that it can be admitted when bids untruthfully, which could only be resulted from that it submitted a lower asking price $s_i' < \tilde{s}_i$ to the central auctioneer at the beginning of the auction.

Note that the primary user N is the original N^{th} seller in the vertex set S. And the seller u_i is in the position behind the seller u_N, *i.e.* $i \geq N$. After seller u_i lies on its asking price, it will jump to the position ahead of that of the seller u_N and the original $(N-1)^{\text{th}}$ seller will be the Nth seller. So the payment after lying will be based on the original $(N-1)^{\text{th}}$ seller's asking price s_{N-1}. Note that the seller u_i itself could also be the $(N-1)^{\text{th}}$ seller after lying. So the utility function after it admitted is:

$$
\begin{aligned}
\mathcal{U}_i'^p &= -V_i(t) + V_i(t + t_i) - \mathbf{p}_i'^P \\
&= -V_i(t) + V_i(t + t_i) \\
&\quad + V_{N-1}(t) - V_{N-1}(t + t_i) \\
&\leq 0
\end{aligned}
$$

The last inequality is based on the proof of Lemma 3. When it cannot be admitted when bids untruthfully, the utility will always be 0. So our mechanism $\mathcal{M}$ is truthful in this case.

Then the lemma follows. ∎

Now we show that our mechanism $\mathcal{M}$ is truthful for the buyer side. Unlike the primary users, the secondary users could

also lie on the required time slots in addition to lying on the bid. Here we use t_i to denote the truthful requested time of secondary user v_i, and t_i' to denote the untruthful requested time. The payment under truthful bidding for secondary user v_i is denoted as $\mathbf{p}_i^s$, and that under untruthful bidding is denoted as $\mathbf{p}_i'^s$. So to prove truthful for the buyer side we need to prove both parts: (1) value-SP, (2) time-SP. We first show that $\mathcal{M}$ is time-SP, then show value-SP.

Lemma 10: Mechanism $\mathcal{M}$ is *time-SP* for secondary users.

Proof: Recall that we assume each request of a secondary user could only claim a longer required time duration than its actual requirement.

Get admitted when truthful: If it can also get admitted when bidding untruthfully, the utility function is:

$$
\begin{aligned}
\mathcal{U}_i'^s &= b_i \cdot t_i - \mathbf{p}_i'^s \\
&< b_i \cdot t_i - b_N \cdot t_N - V_N(t + t_N) + V_N(t + t_i) \\
&= \mathcal{U}_i^s.
\end{aligned}
$$

So the utility $\mathcal{U}_i'^s$ after buyer v_i lies is less than the truthful utility $\mathcal{U}_i^s$.

If it cannot get admitted when bidding untruthfully, our mechanism is trivially truthful under this case for that our mechanism is individual rational for buyers (Theorem 14).

Not admitted when truthful: If it can get admitted when bidding untruthfully, after admitted, the utility is:

$$
\begin{aligned}
\mathcal{U}_i'^s &= b_i \cdot t_i - \mathbf{p}_i'^s \\
&< b_i \cdot t_i + V_N(t + t_i) - (b_N \cdot t_N + V_N(t + t_N)) \\
&= V_N^i(t) - V_N^N(t) \leq 0
\end{aligned}
$$

The last inequality is because that originally buyer v_i cannot be admitted, *i.e.* $V_N^i(t) \leq V_N^N(t)$. So the utility will not increase.

If it cannot get admitted when bidding untruthfully, the utility will remain 0.

So the lemma holds. ■

Next, we show that our mechanism $\mathcal{M}$ is value-SP for secondary users.

Lemma 11: Mechanism $\mathcal{M}$ is *value-SP* for secondary users.

Proof: We prove $\mathcal{M}$ is value-SP for each case in the following:

Get admitted when truthful: If it can get admitted after lying on the bidding price, for that the pricing of $\mathcal{M}$ is bid-independent, so no matter buyer v_i lies or not, the payment $\mathbf{p}_i^s$ is same. So the utility of buyer v_i will not increase after it lies.

If it cannot get admitted after lying on the bidding price, the utility will not increase by the individual rationality of $\mathcal{M}$.

Not admitted when truthful: If it can get admitted when bidding untruthfully, the only possibility is that it submitted a higher bid value $b_i' > \tilde{b}_i$. After admitted, the utility of buyer v_i is:

$$
\begin{aligned}
\mathcal{U}_i'^s &= b_i \cdot t_i - \mathbf{p}_i'^s \\
&= b_i \cdot t_i - (b_N \cdot t_N + V_N(t + t_N) - V_N(t + t_i)) \\
&= V_N^i(t) - V_N^N(t) \leq 0
\end{aligned}
$$

The last inequality is due to that truthfully buyer v_i will not be admitted, *i.e.* $V_N^i(t) \leq V_N^N(t)$. So the utility of buyer v_i will not increase after it lies.

If it cannot get admitted when bidding untruthfully, the utility will remain 0.

So $\mathcal{M}$ is value-SP for buyers. ■

Based on Lemma 10 and Lemma 11, we have proved that our mechanism $\mathcal{M}$ is truthful for the buyers.

Theorem 12: Mechanism $\mathcal{M}$ is strategyproof for both primary users (sellers) and secondary users (buyers).

Next, we demonstrate our mechanism $\mathcal{M}$ is ex-post budget balanced and feasible.

Theorem 13: The mechanism $\mathcal{M}$ is ex-post budget balanced and feasible.

Proof: To establish ex-post budget balance of $\mathcal{M}$, it is enough to prove that the sum of all payments of the winners in each decision phase of the auction is not negative, *i.e.* $\sum_{i=1}^{N-1}(\mathbf{p}_i^s + \mathbf{p}_i^p) \geq 0$. Note that for all admitted primary (secondary) users, we charge the same payment. So we only need to prove the no-deficit property for any winner pair.

$$
\mathbf{p}_i^p + \mathbf{p}_i^s = b_N \cdot t_N + V_N(t + t_N) - V_N(t) \geq 0 \quad (5)
$$

Note that the "$\geq$" relation is resulted from the fact that the bidder N and seller N are matched in the matching bipartite graph $BG_t(B, S, E)$. So our mechanism $\mathcal{M}$ is ex-post budget balanced.

The property of feasible requires that at any time the winning buyers will actually get the goods. In our model, a feasible mechanism needs to satisfy that at any time the winning secondary user will actually be allocated some time of one spectrum channel. It is clear that our mechanism $\mathcal{M}$ is feasible, for that in $\mathcal{M}$ if we admit ONE secondary user, then we must admit a corresponding primary user, *i.e.* we only admit winning (primary user, secondary user) pairs. So the theorem holds. ■

At last, we prove that our mechanism $\mathcal{M}$ is individual rational, *i.e.* the expected utility of any seller is no less than 0 and no buyer pays more than its bid (per time-slot). Individual rationality requires $\mathcal{U}_i^z \geq 0, z \in \{p, s\}$ in each decision phase.

Theorem 14: Mechanism $\mathcal{M}$ is individual rational, *i.e.* in each decision phase, $\mathcal{U}_i^z \geq 0, z \in \{p, s\}$.

Proof: First we prove that our mechanism $\mathcal{M}$ is individual rational for primary user u_i. For that in the computation of $V_i(t)$, $b_i(k)$ is always no less than s_i. By Theorem 12, we know that the dominant strategy of primary user is that $s_i = \tilde{s}_i$, so $b_i(k) \geq \tilde{s}_i$. Thus at each time slot, the expected bidding price is no less than the true valuation of the primary user u_i, so the expected utility of u_i $\mathcal{U}_i^p \geq 0$. So our mechanism $\mathcal{M}$ is individual rational for primary users.

Since the utility function of all the losing secondary users is 0, we only need to consider the utility function of the winning secondary users. We prove that the utility function of admitted secondary users is no less than 0.

$$
\begin{aligned}
\mathcal{U}_i^s &= b_i \cdot t_i - \mathbf{p}_i^s \\
&= b_i \cdot t_i - b_N \cdot t_N - V_N(t + t_N) + V_N(t + t_i) \\
&= V_N^i(t) - V_N^N(t)
\end{aligned}
$$

For that we have done the second sorting, $V_N^i(t) \geq V_N^N(t)$. So $\mathcal{U}_i^s \geq 0$. We finished the proof. $\blacksquare$

V. SIMULATION RESULTS

We conduct extensive simulations to study the performance of our online mechanisms $\mathcal{M}$ for the complete conflict graph model which simulate the situation in urban area.

A. Simulation Setup

In our simulations, we assume there are 10 primary users, and the total available time for the spectrum channel of each primary user is $T = 1000$ time-slots. We generate random secondary requests with random bid values and time requirements. The arrivals of these requests follow Poisson distribution with an arrival rate λ, which varies in our simulations to simulate lightly loaded or heavily loaded system. The asking prices of sellers follow the same probability distribution type of the bidding prices of buyers in our simulations(, for example normal distribution or uniform distribution).

The bidding price and time requirement of each request are either uniformly distributed or normally distributed in our simulations. Here we generate four different sets of requests.

Set 1 The bid value of each request is uniformly distributed in $[0, 1]$, time requirement of each request is uniformly drawn from all integers in $[1, 50]$;

Set 2 The bid value of each request is uniformly distributed in $[0, 1]$, time requirement of each request is uniformly drawn from all integers in $[1, 500]$;

Set 3 The bid value of each request is normally distributed with expected value $\mu_b = 0.5$, and variance $\sigma_b = 2$, time requirement of each request is normally distributed with expected value $\mu_t = 25$, and variance $\sigma_t = 3$;

Set 4 The bid value of each request is normally distributed with expected value $\mu_b = 0.5$, and variance $\sigma_b = 2$, time request of each request is normally distributed with expected value $\mu_t = 250$, and variance $\sigma = 9$.

The bid value are randomly matched and independent with the time requirement.

We also consider the different probability parameters of asking prices and bidding prices in our simulations. We borrow the economic term *Balanced market between supply and demand* to denote that the distributions of asking prices and bidding prices are exactly the same, *i.e.* same probability distribution type with same parameters, and *seller's market* (*resp. buyer's market*) to denote that the probability distribution type of asking prices and bidding prices is the same but the expectation of asking prices is higher (*resp.* lower) than that of bidding prices.

All the simulations is performed 20 times and the result is the averaged value.

B. Performance of Mechanism $\mathcal{M}$

In this section, we study the performance of our mechanism $\mathcal{M}$ compared with the off-line VCG mechanism. We mainly focus on the performance of social efficiency ratio, and total spectrum utilization ratio of our mechanism $\mathcal{M}$. Recall that

when we calculate the expected earning V_i of each primary user i, there is a parameter δ which is used to calculate b_i which is the expected bidding price. Before we study the performance, we need to set a proper value for δ such that the revenue earned by any primary user is no less than the cost of that primary user. We plot result for our 4 different sets of requests, here $\lambda = 10$ equals to the number of primary users and we consider the balanced market between supply and demand.

From Fig. 2 (b), we get that when $\delta = 0.7$ to 0.9 the spectrum utilization ratio is maximized, and when $\delta \geq 0.7$ the revenue earned by any primary user is no less than the cost of that primary user as shown in Fig. 2 (a). We then set $\delta = 0.9$ in our following simulations, because that although we get this value in the simulation which we set $\lambda = 10$ and consider the balanced market, this value is reserved enough to fit for other conditions such as $\lambda \neq 10$ and/or other market.

In Fig. 3 (a), we plot the averaged competitive ratio for the *expected* social efficiency among the 10 channels, when the request arrival rate λ varies. From Fig. 3 we can see that the performance of our mechanism is more than 80% of the VCG mechanism. When the required time is in the same order of the available total time of the spectrum channels as in request set 2 and 4, our mechanism is surprisingly better than VCG in expected social efficiency. The reason is that when the requested time is in the same order with the total available time of the spectrum, as in request set 2 and set 4 the expected required time is 0.25 of the total available time, in the fourth quarter of the available time the expected social efficiency calculated by VCG is very poor, while when the requested time is much less than the total available time, as in request set 1 and set 3 the expected required time is 0.025 of the total available time, only in the last $1/40$ of the available time the expected social efficiency is poor. Given the requests, our mechanism will choose requests that will maximize the social efficiency. So in set 2 and set 4, our mechanism is better than VCG in expected social efficiency. Observe that the competitive ratio of set 2 and set 4 decrease when the arrival rate increases, but the competitive ratio of set 1 and set 3 increase. The reason is that when the arrival rate is big, the competition among requests is high, then our mechanism will converge to the VCG mechanism; When the arrival rate is small, the competition is low, it is harder to admit a request of secondary user in each decision phase. In request set 1 and set 3 the requested time is far less than the total available time which means that the number of decision phases is far more than that for set 2 and set 4, so the social efficiency for request set 2 and set 4 is more than set 1 and set 3.

In Fig. 3 (b), we plot the total spectrum utilization ratio for the four request sets. From Fig. 3 (b), we can see that the spectrum utilization increases when the arrival rate of requests increases. When the system is highly loaded, the spectrum utilization ratio could be as high as 80%, while the spectrum utilization ratio is more than 34% for lightly loaded system. When the system load is low, the competition among requests is low and thus we may not admit feasible requests. In all cases, mechanism $\mathcal{M}$ improve the efficiency ratios without sacrificing the spectrum utilization.
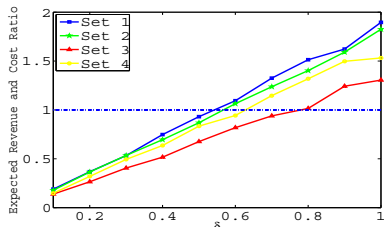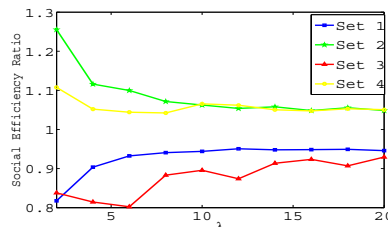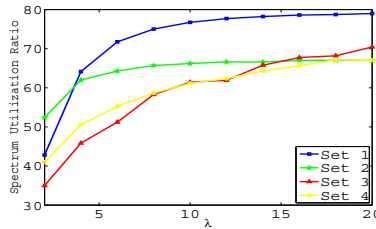
(a) Revenue and Cost Ratio

(a) Social Efficiency Ratio

(a) Social Efficiency Ratio

(b) Spectrum Utilization Ratio

(b) Spectrum Utilization Ratio
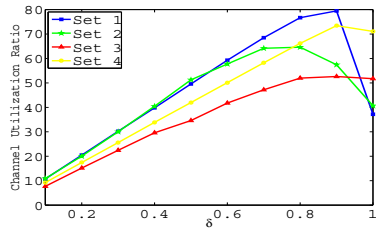
(b) Spectrum Utilization Ratio

Fig. 2. Choose the proper value for δ.

Fig. 3. The performance of $\mathcal{M}$ in balanced market.
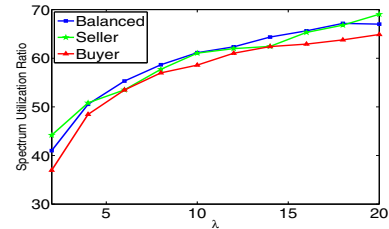
Fig. 4. The performance of $\mathcal{M}$ under request set 4 in different market.

We plot the expected social efficiency ratio and spectrum utilization ratio of request set 4 for different market in Fig. 4. In buyer's market, the expectation μ of asking price is 1.5 times of that of bidding price, while in seller's market, the expectation μ of asking price is 0.667 times of that of bidding price in our simulation setup. From Fig. 4(a), we can see that our mechanism $\mathcal{M}$ performs best in buyer's market and worst in seller's market, which is compared with the VCG mechanism. The reason is that in buyer's market, the expected social efficiency of VCG mechanism is poor for that the asking price is lower than bidding price in expectation, while our mechanism will admit the requests that maximize the expected social efficiency; In seller's market, the expected social efficiency of VCG mechanism is good for that the asking price is high, while given the requests the competition is low, so it is harder for our mechanism to admit the requests. However, even in seller's market in which our mechanism performs worst, the expected social efficiency ratio is still above 80%.

From Fig. 4(b), we can see that for all the different market, the spectrum utilization ratio is above 60% when the system is highly loaded and above 35% when the system is lightly loaded. In the buyer's market where the expectation of asking prices is better than that of bidding prices, the spectrum utilization ratio is the worst among the three cases, which is not intuitive at all. The reason is that in the buyer's market, the spectrum utilization ratios of different spectrum channels are *unfair*, *i.e.* as the asking price increases the spectrum utilization ratio decreases. From the simulation data, we found that for the lowest 2 asking prices, the spectrum utilization ratio is above 80%, but for the highest 3 asking prices, the spectrum utilization ratio is nearly 30%. So although for the lowest 2 to 3 asking prices the spectrum utilization ratio in buyer's market is better than that of seller's market, when we study the total spectrum utilization ratio of all the channels, that in buyer's market is worse than that of the seller's market.

However, the difference among the three cases in spectrum utilization ratio is not significant.

VI. LITERATURE REVIEWS

How to allocate spectrum channels is essentially combinatorial allocation problem, which have been well studied [1], [13]. For example, Yuan *et al.* [24] introduced the concept of a time-spectrum block to model spectrum reservation in cognitive radio networks, and presented both centralized and distributed protocols for spectrum allocation and show that these protocols are close to optimal in most scenarios. In [20], Wu and Tsang studied the distributed multichannel power allocation problem for the spectrum sharing cognitive ratio networks. Ben-Porat *et al.* [3] gave a scheme scheduling decisions on the Cumulative Distribution Function (CDF). Spectrum band auction [6], [8], [17] has also been well studied. However, these results did not guarantee truthfulness.

Truthfulness is a critical factor to attract participation [12]. Many truthful mechanisms have been developed in conventional double auctions, including single-unit [2], [7], [15] and multi-unit double auctions [2], [10]. The majority of these designs follow the idea of McAfees mechanism [15], using the trade reduction to maintain truthfulness.

Xu and Li *et al.* [14] designed efficient methods for various dynamic spectrum assignment problems. They also showed how to design truthful mechanism based on those methods. Xu and Li *et al.* [21], [22] then designed online spectrum auction mechanisms when only secondary users could be selfish. Zhou *et al.* [25] propose a truthful and efficient dynamic spectrum auction system to serve many small players. Although truthfulness is addressed in these works, they all consider single-sided auctions. A recent result by Zhou and Zheng [26] designed truthful double spectrum auctions where multiple parties can trade spectrum based on their individual needs and consider spectrum reuse. However, they did not consider the time domain and their work can not be used in

the online auction. Another recent work by Xu *et al.* [23] designed SALSA for online spectrum admissions. In this work, they considered two cases: Random-Arrival case in which the bid values and requested time durations of secondary requests follow some distributions that can be learned, and Semi-Arbitrary-Arrival case in which the bid values could be arbitrary, but the request arrival sequence is random. For both cases, their method can achieve constant approximation compared to the offline VCG auction in both social efficiency and revenue efficiency. However, this work only considered single-sided auction.

VII. CONCLUSIONS

In this paper we designed mechanisms for online spectrum allocation and double auction when the selfish primary users want to sublease the usage of their spectrum channels for a period of time and many secondary users will bid for the usage in different time-slots. Under a simple assumption that the requests by secondary users arrive with Poisson distribution and the willing payment per time-slot is independent from the time-slot required, we prove that every secondary user will maximize its expected profit if it proposed requests truthfully. We also prove that our mechanism is individual rational for both primary users and secondary users and ex-post budget balanced for the central auctioneer. To the best of our knowledge, this is the first *online* spectrum allocation and double auction protocol with these properties. Our extensive simulation studies show that our protocols perform well.

There are a number of interesting questions that are left for future research. First, we assume that the asking prices of the primary users is fixed during the double auction. However, the asking prices could be adaptive, for example, when the requests are less, the primary users would like to lower their asking price to sell more fraction of the spectrum channels. Second, in our study we assume that the request is not periodic. In practice, the requests of secondary users could be periodic. So how to design a truthful double auction mechanism to fit the period requests is a very interesting problem. We also need to extend our mechanism to the scenarios when the interference could not be modelled as a complete graph.

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