# Wavelength Assignment in WDM Rings to Minimize SONET ADMs

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*Abstract*— **We study wavelength assignment for lightpaths over WDM rings to minimize the SONET ADMs used. This problem has attracted much attention recently. However, its computation complexity remains unknown, and the only known heuristic [6] which does not allow the splitting of lightpaths is problematic in both the algorithm itself and its performance analysis. We first prove the NP-completeness of this problem, followed by a nontrivial randomized $\frac{3+e}{1+e}$ -approximation scheme. We then present a tighter lower bound on the minimum number of ADMs required. After that, we show the incorrectness of the known heuristic and then modify it to make it correct. We also propose three additional heuristics. Their performances are compared through extensive simulation studies.**

*Keywords*— **Wavelength assignment, wavelength division multiplexing (WDM), optical networks, SONET, add-drop multiplexer (ADM).**

## I. INTRODUCTION

WDM ring networks are being deployed by a growing number of telecom carriers to support multiple high-level SONET/SDH self-healing rings over a single physical fiber optical ring. One of the most fundamental network design problems for WDM networks is the assignment of wavelengths to a given set of lightpaths. While most of the previous work attempts to minimize the number of wavelengths required for a given set of lightpaths, or if given a fixed number of wavelengths, to minimize the number of blocked lightpaths [1], [2], [4], [7], [8], [9], it was argued in [5], [6] that unless the wavelength limit is exceeded, the first-order optimization goal should be to minimize the overall network cost which is dominated by the number of required SONET add/drop multiplexers (ADMs) and not the number of wavelengths. It was also shown in [6] that these two minimization problems are intrinsically different, and there exist cases where the two minima cannot be simultaneously achieved.

In [6], a simple lower bound of the number of ADMs is derived. In addition, two heuristics to minimize the number of ADMs were developed: **Cut-First**, and **Assign-First**. The former allows splitting of lightpaths while the latter does not. We further these studies and assume that the lightpaths are not allowed to be split. First of all, despite of efforts in [6] in developing the heuristics, the computation complexity of the problem remains unknown. Thus, the first part of our effort proves the NP-completeness of this problem and discuss the approximality issues. Second, the performance analysis to **Assign-First** is incorrect and we present a counter-example to illustrate this. We modify the algorithm and provide a correct performance analysis. After that, we present three additional greedy heuristics: **Iterative Merging**, **Iterative Matching**, and **Euler Cycle Decomposition**. Due to the difficulty of the theoretical analysis, we conduct extensive simulations to compare the performance of the three heuristics.

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## II. PROBLEM FORMULATION

Assume a Wavelength Division Multiplexed (WDM) ring network consisting of $N$ optical nodes labeled from 0 to $N-1$ in clockwise order. Let $R$ be a set of lightpaths $\{(s_i,t_i) : 1 \leq i \leq |R|\}$ where the lightpath $(s_i,t_i)$ is the arc from $s_i$ to $t_i$ in clockwise direction over the ring. For simplicity, we call $s_i$ the *origin* node and $t_i$ the *termination* node. A wavelength assignment to a set of lightpaths is *valid* if any two overlapping lightpaths are assigned different wavelengths. A valid wavelength assignment to a set of lightpaths $R$ corresponds to a vertex coloring of the *circular arc graph* determined by $R$ as follows: the vertex set is $R$ and there is an edge between two lightpaths if only if these two lightpaths overlap with each other.

It is easier to think of each wavelength around the ring as providing the connectivity for a single SONET ring. Each lightpath within a SONET ring uses two ADMs, one at the origin, the other at the termination. However, two adjacent lightpaths in the same SONET ring, i.e., with the same wavelength, can share one ADM at the common node. Our focus is to address the *minimum ADM problem*: assign wavelengths channels to a given set of lightpaths such that the number of ADMs used is minimized. A closely related problem is the *maximum ADM sharing problem*: assign wavelengths channels to a given set of lightpaths such that the number of ADMs shared by the lightpaths is maximized. It's obvious that these two problems address the same question, indeed, for any wavelength assignment, the sum of the number of ADMs used and the number of ADMs shared is always equal to twice the number of lightpaths.

We define a *segment* to be a sequence of lightpaths in which the termination of a lightpath (except the last one) is the origin of the subsequent lightpath, and any two lightpaths in the sequence do not overlap with each other. A segment is said to be a *circle segment* if the termination of the last lightpath is also the origin of the first lightpath, in other words, a circle segment covers the entire ring. A segment which is not a circle segment is said to be a *noncircle segment*. The number of ADMs to be used by a circle segment is equal to the number of lightpaths inside the segment. The number of ADMs to be used by a noncircle segment is one more than the number of lightpaths inside the segment. Thus the total number of ADMs used is the number of noncircle segments plus the total number of lightpaths. Thus the minimum ADM problem can be solved in two phases: in the first phase, the lightpaths are grouped into segments such that the number of noncircle segments is as small as possible, in the second phase, assign the wavelength channels to the segments. Note that the second phase is exactly the well-studied circular-arc coloring problem. It only affects the number of wavelengths used, but has no impact on the number of ADMs used.

## III. Computational Complexity

Although some heuristics were presented in [6] to the minimum ADM problem, the computational complexity of this optimization problem remains open. In this section, we show that the problem is NP-complete.

Let $R$ be any set of lightpaths. $R$ is said to be *uniform* if the number of lightpaths passing through each link is the same. In particular, if there are exactly $L$ lightpaths in $R$ passing through each link, $R$ is said to be $L$-*uniform*. We have the following result on the NP-completeness of the minimum ADM problem.

*Lemma 1:* The minimum ADM problem is NP-complete even if the set of lightpaths is restricted to be uniform.

*Proof.* We reduce the circular-graph coloring problem to the minimum ADM problem. The circular-arc coloring problem has been proven to be NP-complete in [3]. The proof in [3] actually implies the following stronger result:

Given an $L$-uniform lightpath set $R$, to decide whether its corresponding circular arc graph is $L$-colorable is *NP-Complete.*

Let $R$ be any $L$-uniform lightpath set. Note that the chromatic number of the circular-arc graph corresponding to $R$, denoted by $\chi(R)$, is at least $L$, and $\chi(R) = L$ if and only if $R$ can be partitioned into $L$ subsets with the lightpaths in each subset form a ring. On the other hand, the minimum ADMs required by any $Lopt(R) = |R|$ if and only if $R$ can be partitioned into $L$ subsets with the lightpaths in each subset form a ring. Thus $opt(R) = |R|$ if and only if $\chi(R) = L$. This implies that the minimum ADM problem is NP-complete even if the lightpath set is constrainted to be uniform.

In the next section, we provide a tighter lower bound and heuristic algorithms for the minimum ADM problem.

## IV. $\frac{3+e}{1+e}$−Approximating Scheme

We exploit integral multicommodity flow model. By $L$ we denote the maximal link load (assuming) at link 0, $R_1 \subset R$ is the set of lightpaths containing link 0, $R_2 := R \backslash R_1$. The number of wavelengths used for optimal solution is between $L$ and $|R|$. Assume it is $W$. We add $W-L$ one-hop "fake" lightpaths at link 0 and cut the $W$ lightpaths containing link 0 at its midpoint and then wire the ring to a line. Assume $X = \{x_i | 1 \leq i \leq W\}$ is the set of all right halves of cut lightpaths and $Y = \{y_i | 1 \leq i \leq W\}$ is the set of left halves. By $X_1 := \{x_i | L+1 \leq i \leq W\}$, $Y_1 := \{y_i | L+1 \leq i \leq W\}$ we denote all "fake" right and left halves. We build an acyclic digraph $G$ with unitary vertex capacity and edge cost function $c$ as follows. $G = (V, E)$, where $V = R_2 \cup X \cup Y$. For any $r_1 \in V, r_2 \in V$, if $r_1$ and $r_2$ do not overlap and $\{r_1, r_2\} \nsubseteq X_1 \cup Y_1$ and $r_1$ is at the left of $r_2$ then we add an edge $(r_1, r_2)$.

Now define $c$, $\forall (r_1, r_2) \in E$,

Case 1. $r_1, r_2$ share some endpoint. Assign cost 1.

Case 2. $r_1, r_2$ do not share endpoint, we have two sub cases.

Case 2.1 $\{r_1, r_2\} \cap (X_1 \cup Y_1) = \Phi$, assign cost 2.

Case 2.2 $\{r_1, r_2\} \cap (X_1 \cup Y_1) \neq \Phi$, assign cost 1.

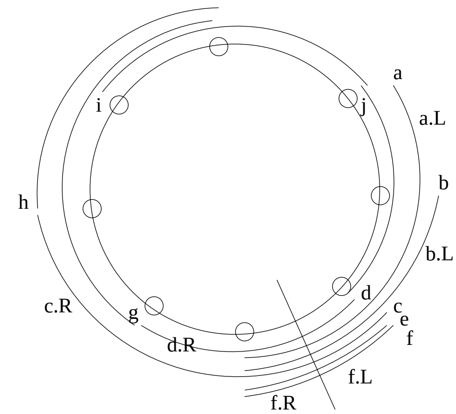


Fig. 1. Cutting. $L = 4, W = 6$. $e, f$ are "fake" lightpaths.

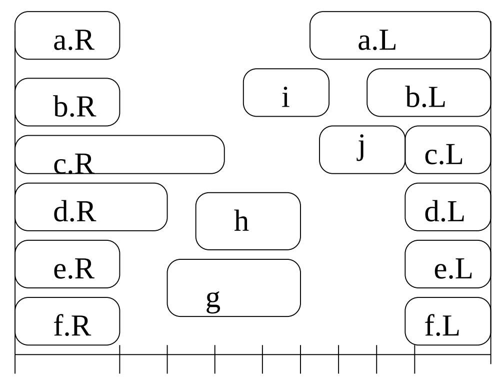


Fig. 2. Wire to a line. $X = \{a.R, b.R, ..., f.R\}$, $Y = \{a.L, b.L, ..., f.L\}$, $X_1 = \{e.R, f.R\}, Y_1 = \{e.L, f.L\}$

Any feasible solution is viewed as a set of $W$ vertex disjoint paths in $G$ that links all $W$ pairs $\{(x_i, y_i) | 1 \leq i \leq W\}$ and contains all vertices and each path is corresponding to a unique wavelength. Thus, the two halves of any cut lightpath are assigned the same wavelength, and our approach finally does not cut lightpaths. For any feasible solution, if walking from all $x_i$'s to arrive at all $y_i$'s along such $W$ paths, the total edge cost we collect is exactly the number of ADMs needed. So the integral multicommodity model (Please refer to textbooks for this ILP formulation) is:

To minimize

$$z = \sum_e \sum_{1 \leq i \leq W} f_e^i c_e \tag{1}$$

Subject to:

$$\sum_{i, e: e=(r,v)} f_e^i = 1, \forall r \in R_2 \tag{2}$$

$$\sum_{i, e: e=(v,r)} f_e^i = 1, \forall r \in R_2 \tag{3}$$

$$\sum_{e: e=(x_i, v)} f_e^j = \delta_{i,j}, i, j = 1, ..., W \tag{4}$$

$$\sum_{e:e=(v,y_i)} f_e^j = \delta_{i,j}, i,j = 1,...,W \quad (5)$$

$$f_e^i \in \{0,1\}, i = 1,...,W., \forall e. \quad (6)$$

Here

$$\delta_{i,j} = 1, i = j; \delta_{i,j} = 0, i \neq j. \quad (7)$$

By relaxing the constraints (6) as

$$0 \leq f_e^i \leq 1, i = 1,...,W, \forall e. \quad (6')$$

and solving the relaxed linear programming problem, we get a feasible fractional $W$-commodities flow $F^*$ with objective value $z^*$ . We decompose the $i$-th commodity flow into $K_i$ paths from $x_i$ to $y_i$. Assume its $j$-th path is $P_{i,j}$ and it carries flow $f^{i,j}$.

We manafacture $W$ dice. The $i$-th die has $K_i$ faces, and its $j$-th face is assigned probability $f^{i,j}$ of being selected. We flip all these $W$ dice. If we get $j$-th face selected on the $i$-th die, we select path $P_{i,j}$. Use $P_i$ to denote the $i$-th such selected path. We get a "coarse" integral multicommodity flow $\widetilde{F} = (\widetilde{f_e^i})_{1\leq i\leq W, e\in E}$ consisting of $W$ paths $P_1, P_2, ..., P_W$ with objective value $\tilde{z}$. Here $E[\tilde{z}] = z^*$. Easy to check, $\widetilde{F}$ satisfies constraints in (4)(5)(6) but may not satisfies constraints (2)(3).

By $U \subset R$ we denote $V(P_1)\cup V(P_2)\cup ...\cup V(P_W)\cup R_1 \backslash (X \cup Y)$, here $V(P_i)$ is the set of vertices of the path $P_i$. We assign the $i$-th wavelength to the lightpaths in $P_i$. This assignment uses $\tilde{z}$ ADMs for the lightpath set $U$.

$\forall r \in R_2$, (2)(3) are both satisfied or neither is satisfied by our coarse solution $(\widetilde{f_e^i})$, since the left of two constraints are always equal. When flipping the $i$-th die, $\forall\ r \in R_2$, with probability

$$p_{r,i} := \sum_{j:r\in P_{i,j}} f^{i,j}$$

$r \in P_i$. Furthermore we have

$$\sum_{1\leq i\leq W} p_{r,i} = 1 \quad (8)$$

Since all $W$ flipping experiments are independent, using (8) one can prove

$$Pr(r \notin U) = (1-p_{r,1})...(1-p_{r,W}) < \frac{1}{e}$$

So

$$E[|R\backslash U|] < \frac{1}{e}(|R_2|)$$

For any request not in $U$, we assign an individual wavelength and 2 ADMs. Assume totally we used $S$ ADMs and the optimal solution uses Opt ADMs. We have

$$\begin{aligned} E[S] &\leq E[\tilde{z}] + 2E[|R_2|] \\ &< z^* + \frac{2}{e}(|R| - L) \\ &\leq Opt + \frac{2}{e}(|R| - L) \\ &\leq Opt + \frac{2}{e}(Opt - L) \end{aligned}$$

Combined with the trivial result (Please refer to section VI) $S \leq Opt + 2L$, we get a probability algorithm with solution $S$ satisfying

$$E[S] < \frac{3+e}{1+e} Opt$$

This ratio is better than the trivial ratio of 2 (each request uses one individual wavelength and two ADMs).

The randomized approach in fact gives a solution space, for which the average worst case ratio is $\frac{3+e}{1+e}$. But we may have discarded many terms when estimating the worst ratio, for example, $\Pr(r \notin U)$ may be far less than $\frac{1}{e}$ when $p_{r,1},...,p_{r,w}$ are not equally distributed. So for certain instances, we can do the experiments several times and select the best results which might give ratio better than the worst ratio.

Another remaining problem is that we assume we know $W$, the number of wavelengths that the optimal solution uses. It is not a problem if we call the above procedure several times by setting $W = L, L+1, ..., |R|$ and select the best result we get, since the value of $W$ that the optimal solution used should be in the interval $[L, |R|]$. So finally we get the polynomial time probability algorithm with worst expected ratio $\frac{3+e}{1+e}$.

## V. Tighter Lower Bound

Let $\sigma_i$, $\tau_i$ and $x_i$ denote the total number of lightpaths originating at, terminating at, and crossing over node $i$ respectively. Let $\ell_i$ denote the load on the link between node $i$ and node $i+1$. A simple and straightforward lower bound on the minimum number of ADMs is given in [6]:

$$\alpha_{lb} = \sum_{i=0}^{N-1} \max(\sigma_i, \tau_i)$$

The calculation of this lower bound ignores the fact that the two lightpaths can be matched only if they do not overlap with each other. In this section, we provide a stronger lower bound based on the maximum matching.

At each node $i$, we construct a bipartite graph $H_i = (U_i, V_i, E_i)$, where

- $U_i$ is the set of lightpaths ending at node $i$;
- $V_i$ is the set of lightpaths starting from node $i$;
- for any $u \in U_i$ and $v \in V_i$, $(u,v) \in E_i$ if and only if $u$ and $v$ do not overlap with each other.
- $m_i$ is the size of the maximal matching of $H_i$.

Then after considering the constraint that any two lightpaths can not overlap if they share an ADM, we have the following lower bound for the number of SONET ADMs we need:

$$\begin{aligned} \alpha'_{lb} &= \sum_{i=0}^{N-1} (\sigma_i + \tau_i - m_i) \\ &= \sum_{i=0}^{N-1} (\sigma_i + \tau_i) - \sum_{i=0}^{N-1} m_i \\ &= 2\,|R| - \sum_{i=0}^{N-1} m_i. \end{aligned}$$

It should be noted that the above lower bound is stronger than the lower bound $\alpha_{lb}$, because

$$\begin{aligned}\alpha_{lb} &= \sum_{i=0}^{N-1} \max(\sigma_i, \tau_i) \\ &= \sum_{i=0}^{N-1} (\sigma_i + \tau_i - \min(\sigma_i, \tau_i))\end{aligned}$$

and $\min(\sigma_i, \tau_i) \geq m_i$.

## VI. **Assign First: Revisited**

If all lightpaths do not cross over some link, then the lightpaths form an interval graph, and the minimum ADM problem for such instance has a greedy solution with polynomial time complexity. Based on this conclusion, the **Assign First** heuristic presented in [6] initially assign all lightpaths that pass through a carefully selected link with unique wavelengths, and then use the above greedy approach to assign wavelengths to the remaining lightpaths. We found some error in its analysis and modified it to avoid the error.

The **Assign First** heuristic presented in [6] initially assign all lightpaths that pass through a carefully selected link with unique wavelengths, and then greedily assign wavelengths as above to the remaining lightpaths, which form an interval graph. It was shown in [6] that the number of ADMs used by **Assign First** is at most $\sum_{i=0}^{N-1} \max\{\rho_i, \tau_i\} + \min_{i=0}^{N-1}\{x_i + \min\{\rho_i, \tau_i\}\}$

Before we explain why the analysis and the algorithm itself are incorrect, we first show by a counter-example that above expression is incorrect even as an upper bound. Consider a ring network with $N = 4n$ nodes, numbered from 0 to $N-1$ in the clockwise direction. A set of $M = N$ lightpaths is defined as follows: $T = \{(i, (i + 2n + 1) \mod N) \mid 0 \leq i < N\}$ Each lightpath traverses $2n + 1$ links, which is more than half way around the ring. Thus, all these lightpaths must be assigned different wavelengths. This means no ADMs can be shared. Each lightpath requires 2 ADMs, and the total number of ADMs used is $2M = 8n$. On the other hand, $\sigma_i = \tau_i = 1$ and $x_i = 2n$. According to the above upper bound, the total number of ADMs used by Assign-First is at most $N + 2n + 1 = 6n + 1$, which is impossible. Thus the performance analysis of the **Assign-First** given in [6] must be incorrect.

Consider any $0 \leq i < N$. Let $R_i$ denote the set of links passing through the link $i$. We first greedily assign wavelengths to the lightpaths not in $R_i$ and assume we have used $W_i$ wavelengths. After that we construct a weighted bipartite graph $G_i = (R_i, W_i, E_i)$: There is an edge between a lightpath $(s, t) \in R_i$ and a wavelength $w \in W_i$ if and only if

- the lightpath $(s, t)$ does not overlap with any lightpath assigned with $w$,
- either $s$ is the termination node of some lightpath assigned with $w$, or $t$ is the origin node of some lightpath assigned with $w$.

Suppose there is an edge between $(s, t) \in R_i$ and $w \in W_i$. If $s$ is the termination node of some lightpath assigned with $w$ *and* $t$ is the origin node of some lightpath assigned with $w$, then the weight of the edge is set to 2, otherwise 1. The weight defined this way represents the number of ADMs shared if the lightpath $(s, t)$ is assigned with $w$. Find a maximum-weighted matching in $G_i$. Each edge in the matching induces a wavelength assignment to a lightpath in $T_i$. If there are more lightpaths in $R_i$ not assignde by the matching, we assign each of them a unique wavelength . We do this procedure for all $0 \leq i < N$, and selet the best. Easy to check, the number of ADMs used is at most

$$\begin{aligned}&\sum_{i=0}^{N-1} \max\{\sigma_i, \tau_i\} + 2\min\{\ell_i : 0 \leq i < N\} \\ \leq\ & Opt + 2L\end{aligned}$$

## VII. Greedy Segmenting Approaches

As indicated in Section II, the minimum ADM problem can be solved in two phases: in the first phase, the lightpaths are grouped into segments such that the number of noncircle segments is as small as possible; in the second phase, assign the wavelength channels to the segments. The first phase completely determines the number of ADMs used, which is equal to the number of noncircle segments plus the total number of lightpaths. The second phase intends to minimize the wavelength usage. In this section, we present three general greedy approaches for the first phase, **Iterative Merging**, **Iterative Matching**, and **Euler Cycle Decomposition**.

### *A. Iterative Merging*

Initially we have $|R|$ segments, with each segment consisting of one lightpath. At each step, one of the following three possible operations is performed in decreasing priority:

*Operation 1.* Merge two noncircle segments into a circle segment.

*Operation 2.* Split a noncircle segment into two noncircle segment and then merge one of them with another noncircle segment into a circle segment.

*Operation 3.* Merge two noncircle segments into a larger noncircle segment.

Operation 1 decreases the number of noncircle segments by two, and Operation 2 and Operation 3 both decrease the number of noncircle segments by one. Thus, the algorithm terminates after at most $|R| - 1$ steps.

### *B. Iterative Matching*

The **Iterative Matching** is inspired by the approach to derive the tighter lower bound in Section V. Initially we have $|R|$ segments, with each segment consisting of one lightpath. At each step, at each node $i$ we construct a bipartite graph $G_i = (U_i, V_i, E_i)$, where

- $U_i$ is the set of segments ending at node $i$;
- $V_i$ is the set of segments starting from node $i$;
- for any $u \in U_i$ and $v \in V_i$, $(u, v) \in E_i$ if and only if $u$ and $v$ do not overlap with each other.

We find the maximum matching of $G_i$. Then we pick the node at which the size of the maximum matching is the largest, and merge the segments according to the maximum matching at this node. This procedure is repeated until no matching can be found any more. It's obvious that this algorithm has polynomial run-time.

### C. *Eulerian Circuit Decomposition*

We first show that $\sigma_i = \tau_i$ at any node $i$ if and only if $R$ is uniform. The sufficient part is intuitice and thus we only need to show the necessary part. Suppose that $\sigma_i = \tau_i$ at any node $i$. construct a directed multigraph $G_R$ as follows: $G_R$ has the same node set as the ring and corresponds to a lightpath $(s, t)$, there is a link from $s$ to $t$ in $G_R$. Then $G_R$ is an Eulerian digraph and each of its connected component is also Eulerian. According to Euler's Theorem, each connected component contains an Eulerian circuit. Let $R_1, R_2, \cdots, R_K$ be the partition of $R$ corresponding to the connected components of $G_R$. Then each $R_k$ is uniform and so is $R = R_1 \cup R_2 \cup \cdots \cup R_K$.

Now let $R$ be any uniform set of lightpaths and $R_1, R_2, \cdots, R_K$ be the partition of $R$ corresponding to the connected components of $G_R$. If a $R_k$ is 1-uniform, then the total number of ADMs used required by $R_k$ is exactly $|R_k|$. Now we assume that $R_k$ is $L$-uniform for some $L > 1$. An Eulerian circuit over $R_k$ can be found in polynomial time. This Eulerian circuit can be further decomposed into a number of segments by walking from an arbitrary node in this circuit and generating a segment when there is an overlap. To minimize the number of noncircle segments, we can find the best starting point by enumerating all possible starting points. If all lightpaths in $R_k$ are short in the sense that no $D_k$ of them cover the entire ring, then any Eulerian circuit over $R_k$ can be decomposed into at most $\left\lceil \frac{|R_k|}{D_k} \right\rceil$ segments, and thus we have the following bound on the total number of ADMs used.

*Lemma 2:* Suppose that $R$ is uniform. Let $R_1, R_2, \cdots, R_K$ be the partition of $R$ corresponding to the connected components of $G_R$ and let

$$I = \{1 \le k \le K : R_k \text{ is not 1-uniform}\}.$$

Suppose that for any $k \in I$, no set of $D_k$ lightpaths in $R_k$ cover the entire ring. Then the total number of ADMs used by the Eulerian circuit decomposition heuristic is at most

$$|R| + \sum_{k \in I} \left\lceil \frac{|R_k|}{D_k} \right\rceil.$$

If $R$ is not uniform, we can add

$$\frac{1}{2} \sum_{i=0}^{N-1} |\sigma_i - \tau_i|$$

lightpaths to $R$ to form a uniform set of lightpaths, denoted by $R'$. Then

$$\begin{aligned} |R'| &= |R| + \frac{1}{2} \sum_{i=0}^{N-1} |\sigma_i - \tau_i| \\ &= \sum_{i=0}^{N-1} \max\{\sigma_i, \tau_i\} = \alpha_{lb}. \end{aligned}$$

The **Euler Cycle Decomposition** heuristic can then be applied to $R'$.

## VIII. Simulation of Heuristics

It is hard to obtain the tight performance analysis of the proposed heuristics. Even though we have provided some upper bounds on the number of ADMs by some heuristics in the worst case, their practical and average behaviors are still unknown. In this section, we compare their performance through simulation studies. The underlying ring network consists of $N = 16$ node (16 is recommended to be the maximal number of nodes for SONET rings). The number of lightpaths, $|R|$, is randomly generated between $16$ and $256$. The origin of each lightpath is also randomly generated between $0$ and $16$, and so is the termination of each lightpath. For each instance, we run all three heuristics: **Modified Assign First (MAF), Iterative Matching (IMat)** and **Iterative Merging (IMer)**. Table I lists the $18$ outputs of randomly generated non-trivial experiments. The last row in Table I summarizes the cumulative ADM sharings of the 200 experiments we conducted.

| MAF | IMat | IMer |
|---|---|---|
| 30 | 37 | 43 |
| 13 | 16 | 18 |
| 25 | 29 | 35 |
| 43 | 54 | 67 |
| 39 | 45 | 54 |
| 27 | 33 | 35 |
| 15 | 19 | 19 |
| 10 | 10 | 11 |
| 48 | 67 | 74 |
| 39 | 48 | 60 |
| 31 | 38 | 44 |
| 26 | 34 | 30 |
| 48 | 66 | 74 |
| 47 | 60 | 63 |
| 16 | 22 | 21 |
| 35 | 39 | 49 |
| 44 | 61 | 70 |
| 35 | 41 | 49 |
| 4950 | 6184 | 6922 |

TABLE I

ADM Savings in 18 Experiments.

As demonstrated by Table I, in general, **Iterative Merging (IMer)** outperforms **Iterative Matching** which further outperforms **Modified Assign First.** However this is not always true. For each heuristic, there does exist some instance in which it outperforms the other two.

## IX. Conclusion

We studied the minimum ADM problem which addresses wavelength assignment for lightpaths over WDM rings to minimize the SONET ADMs. We first proved its NP-completeness and gave a randomized solution. We then present tighter lower bounds on the minimum number of ADMs required by any instance of lightpaths. After that, we modified the approach **Assign First**, which is originally presented in [6]. We also proposed three new heuristics. Their performances are compared through extensive simulation studies.

Recently we found the technique to prove: (1) **Iterative Merging** gives approximating ratio 1.75; (2)**Iterative Matching** gives ratio $\frac{5}{3}$; (3) The **Circle Segment First**, modified from iterative matching, in which we greedily find any possible circle segment by depth-first or width-first searching, gives ratio $1.5$.

## X. ACKNOWLEDGEMENT

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