Quality Guaranteed Localized Routing for Wireless Ad Hoc Networks

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***Abstract*—We consider a wireless ad hoc network consisting of n points randomly distributed in a two-dimensional plane. We show that, with high probability, we can locally find a path for any pair of nodes such that the length of the path is no more than a constant factor of the minimum. By assuming each node knows its position, the method decides where to forward the message purely based on the positions of current node, its neighbors, and the positions of the source and the target. Our method is based on a novel structure called localized Delaunay triangulation [1] and an efficient localized routing method [2] that guarantees that the distance traveled by the packets is no more than a small constant factor of the minimum when the Delaunay triangulation of wireless nodes are known.**

Our experiments show that the delivery rates of existing localized routing protocols are increased when localized Delaunay triangulation is used instead of several previously proposed topologies, and the localized routing protocol based on Delaunay triangulation works well in practice. We also conducted extensive simulations of another localized routing protocol, FACE method [3]. The path found by this protocol is also reasonably good compared with previous one although it cannot guarantee a constant approximation on the length of the path traveled theoretically.

I. Introduction

One of the central challenges in the design of *ad hoc* networks is the development of dynamic routing protocols that can efficiently find routes between two communication nodes. In recent years, a variety of routing protocols [4], [5], [6], [7], [8], [9] targeted specifically for *ad hoc* environment have been developed.

Several researchers proposed a set of routing protocols, namely the localized routing, which select the next node to forward the packets based on the information in the packet header, and the position of its local neighbors. Bose and Morin [2] showed that several localized routing protocols guarantee to deliver the packets if the underlying network topology is the Delaunay triangulation of all wireless nodes. They also gave a localized routing protocol based on the Delaunay triangulation such that the total distance traveled by the packet is no more than a small constant factor of the distance between the source and the destination. However, it is expensive to construct the Delaunay triangulation in a distributed manner, and routing based on it might not be possible since the Delaunay triangulation can contain links longer than the transmission radius of the wireless devices. Then, several researchers proposed to use some *planar* topologies that can be constructed efficiently in a distributed manner. Bose *et al.*[3] and Karp *et al.* [10] proposed to use the Gabriel graph [11]. Routing according to the right hand rule, which guarantees delivery in planar graphs [2], is used when simple greedy-based routing heuristics fail.

Using Gabriel graph although can guarantee the delivery of the packets with the help of the right-hand rule, however, the distance traveled by the packet could be much larger than the minimum required [12], [13], [14], [15]. In other words, Gabriel graph is not a good approximation of the unit disk graph in terms of the pair-wise distance between wireless nodes. This is true even when the points are randomly and uniformly distributed in a unit square [12]. Formally, given a graph H, a spanning subgraph G of H is a t-spanner if the length of the shortest path connecting any two nodes in G is no more than t times the length of the shortest path connecting them in H. In [1], Li *et al.* designed a localized algorithm that constructs a planar t-spanner for the unit-disk graph $UDG(V)$, such that some of the localized routing protocols can be applied on it. They obtained a value of approximately 2.5 for the constant t. They called the constructed graph planarized *local Delaunay triangulation* [1], denoted by $PLDel$.

Applying the routing methods proposed in [3], [10] on the planarized localized Delaunay graph $PLDel$, a better performance is expected because the localized Delaunay triangulation is denser compared to the Gabriel graph, but still with $O(n)$ edges. However, these two methods do not guarantee that the ratio between the distance traveled by the packets to the minimum possible. The method proposed by Bose and Morrin [2] does guarantee this distance ratio, but that needs the construction of the Delaunay triangulation, which cannot be constructed and updated efficiently in a distributed manner.

Hence, we are interested in studying the performances of several routing protocols on localized Delaunay triangulation. We prove that the localized Delaunay triangulation almost surely contains the Delaunay triangulation of a set n of randomly distributed wireless nodes when the transmission range r_n satisfies $n\pi r_n^2 \geq 4\frac{\ln n + c(n)}{n}$, where $c(n) \rightarrow \infty$ as n goes infinity. Notice that, Gupta and Kumar [16] showed that the unit disk graph is connected with high probability if the transmission range r_n satisfies $\pi \cdot r_n^2 \geq \frac{\ln n + c(n)}{n}$ for any $c(n)$ with $c(n) \rightarrow \infty$ as n goes infinity. When the unit disk graph is connected, then with high probability, we can construct the Delaunay triangulation $Del(V)$ by constructing the local Delaunay triangulation instead.

We study the performance of the localized routing method by some simulations in which results show the delivery is guaranteed and the ratio of the length traveled by packet to the minimum is small. Our simulations show that the delivery rates of several localized routing protocols are also increased when the localized Delaunay triangulation is used. In our experiments, several simple local routing heuristics, applied on the localized Delaunay triangulation, have always successfully delivered the packets, while other heuristics were successful in over 90% of the random instances. Moreover, because the constructed topology is planar, a localized routing algorithm using the right hand rule guarantees the delivery of the packets from source node to the destination when simple heuristics fail. The experiments also show that several localized routing algorithms (notably, compass routing [17] and greedy routing) also result in a path whose length is within a small constant factor of the shortest path; we already know such a path exists since the localized Delaunay triangulation is a t-spanner.

The remaining of the paper is organized as follows. In Section II, we review some definitions, some related geometry structures, and previously known localized routing protocols for wireless networks. We then show a fully localized routing algorithm that, with high probability, guarantees that the distance traveled by the packets is no more than a small constant factor of the minimum in Section III. We study the performance of the localized routing algorithm based on Delaunay triangulation and various routing protocols on various structures in Section IV. Section V gives a brief conclusion of our paper.

II. Network Model and Preliminaries

We assume that all wireless nodes are given as a set V of n nodes in a two dimensional space. Assume that all wireless nodes have distinctive identities and each static wireless node knows its position information, either through a low-power Global Position System (GPS) receiver or through some other way. Most of our results actually only requires that each node knows the relative positions of its neighbors, which can be achieved by using the angle of arrival of the signal or the strength of the signal. For simplicity, we also assume that all n wireless nodes have the same maximum transmission range, denoted by r_n. We normalize it to one unit if no confusion is caused. By a simple broadcasting, each node u can gather the location information of all nodes within its transmission range. Consequently, all wireless nodes V together define a unit-disk graph $UDG(V)$, which has an edge uv only if the Euclidean distance $\|uv\|$ between u and v is less than one unit. We also use $G(V, r_n)$ to denote such induced unit disk graph. Hereafter, a *broadcast* means a node sends out a message which will be received by all nodes within its transmission range.

Let $\varrho_G(u, v)$ be the path found by a unicasting routing method ϱ from node u to v in a weighted graph G, and $\|\varrho_G(u, v)\|$ be the length of the path. The spanning ratio achieved by a routing method ϱ is defined as $\max_{u,v} \|\varrho_G(u, v)\|/\|uv\|$. Notice that the spanning ratio achieved by a specific routing method could be much larger than the spanning ratio of the underlying structure. Nonetheless, a structure with a small spanning ratio is necessary for some routing method to possibly perform well.

We also assume that there are no four nodes of V that are co-circular. A triangulation of V is a *Delaunay triangulation*, denoted by $Del(V)$, if the circumcircle of each of its triangles does not contain any other nodes of V in its interior. The *Voronoi region*, denoted by $Vor(p)$, of a node p in V is the collection of two dimensional points such that every point is closer to p than to any other node of V. The *Voronoi diagram* for V is the union of all Voronoi regions $Vor(p)$, where $p \in V$. The Delaunay triangulation $Del(V)$ is also the dual of the Voronoi diagram: two nodes p and q are connected in $Del(V)$ if and only if $Vor(p)$ and $Vor(q)$ share a common boundary. The boundary segment of a Voronoi region is called the *Voronoi edge*. The intersection point of two Voronoi edge is called the *Voronoi vertex*. Each Voronoi vertex is the circumcenter of some Delaunay triangle. It is well-known that the Delaunay triangulation $Del(V)$ is a planar t-spanner of the completed Euclidean graph [18], [19].

For convenience, let $disk(u, v)$ be the closed disk with diameter uv, $disk(u, v, w)$ be the circumcircle defined by the triangle $\triangle uvw$, and $B(u, r)$ be the circle centered at u with radius r. Let $x(v)$ and $y(v)$ be the value of the x-coordinate and y-coordinate of a node v respectively.

The following structures were defined for a point set, but here we consider them for UDG. The *relative neighborhood graph*, denoted by $RNG(V)$, consists of all edges uv such that $\|uv\| \leq 1$ and there is no point $w \in V$ in $B(u, \|uv\|) \cap B(v, \|uv\|)$. See [20]. The *Gabriel graph*, denoted by $GG(V)$, consists of all edges uv such that $\|uv\| \leq 1$ and $disk(u, v)$ does not contain any node from V. See [11]. Bose *et al.* [12] showed that the length stretch factor of $RNG(V)$ is at most $n - 1$ and the length stretch factor of $GG(V)$ is at most $\frac{4\pi\sqrt{2n-4}}{3}$. It was shown in [12] that the spanning ratio of Gabriel graph on a uniformly random n points set in a square is almost surely at least $O(\sqrt{\log n/\log\log n})$.

Let $N_k(u)$ be the set of nodes of V that are within k hops distance of u in the unit-disk graph $UDG(V)$. A node $v \in N_k(u)$ is called the k-neighbor of the node u. A distributed algorithm is a *localized algorithm* if it uses only the information of all k-local nodes of each node plus the information of a constant number of additional nodes. A graph G can be constructed locally in the ad hoc wireless environment if each wireless node u can compute the edges of G incident on u by using only the location information of all its k-local nodes.

Assume a packet is currently at node u, and the destination node is t. Several localized routing algorithms that just use the local information of u to route packets (i.e., find the next node v of u) were developed. Kranakis *et al.* [17] proposed to use the compass routing, which basically finds the next relay node v such that the angle $\angle vut$ is the smallest among all neighbors of u in a given topology. Lin *et al.* [21], Bose *et al.* [3], and Karp *et al.* [10] proposed similar greedy routing methods, in which node u forwards the packet to its neighbor v in a given topology which is closest to t. Recently, Bose *et al.*[22], [2], [3] proposed several localized routing algorithms that route a packet from a source node s to a destination node t.

When the underlying network topology is a planar graph, the right hand rule is often used to guarantee the packet delivery after simple localized routing heuristics fail [3], [21], [10]. Morin proved the following results in [23]. The greedy routing guarantees the delivery of the packets if the Delaunay triangulation is used as the underlying structure. The compass routing guarantees the delivery of the packets if the regular triangulation is used as the underlying structure. Delaunay triangulation is a special regular triangulation. There are triangulations (not Delaunay) that defeat these two schemes. The greedy-compass routing works for all triangulations, i.e., it guarantees the delivery of the packets as long as there is a triangulation used as the underlying structure. They proved this by showing that the distance from the selected forwarding node v to the destination node t is less than the distance from current node u to t. However, the same proof cannot be carried over when the network topology is Yao graph, Gabriel graph, relative neighborhood graph, and the localized Delaunay triangulation.

Although some of the localized routing protocols guarantee the delivery of the packet if some special geometry structures are used, none of these guarantees the ratio of the distance traveled by the packets over the minimum possible. Bose and Morrin [2] proposed a method to bound this ratio using the Delaunay triangulation. Notice that constructing Delaunay triangulation in a distributed manner is communication expensive.

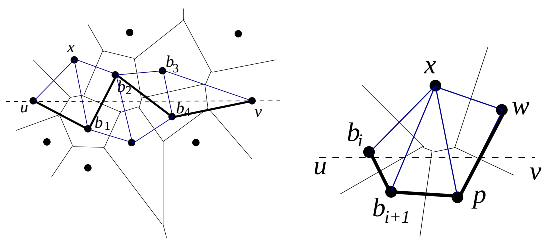
III. Localized Routing Works

A. Routing Based on Delaunay

Bose and Morrin [2] have proposed a method to route the packets using the Delaunay triangulation. Their routing strategy is based on a remarkable proof by Dobkin, Friedman and Supowit [18] that the Delaunay triangulation is a spanner. However, there are plenty of technique details left to be discussed. In this section, we present a completed localized routing method using the Delaunay triangulation.

To discuss the localized routing algorithm, we need a quick review of the proof by Dobkin, Friedman and Supowit [18]. They proved that the Delaunay triangulation is a t-spanner by constructing a path $\Pi_{dfs}(u, v)$ in $Del(V)$ with length no more $\frac{1+\sqrt{5}}{2}\pi\|uv\|$. The constructed path consists of at most two parts: one is some *direct DT* paths, the other is some *shortcut* subpaths.

Given two nodes u and v, let $b_0 = u$, b_1, b_2, $\cdots$, b_{m-1}, $b_m = v$ be the nodes corresponding to the sequence of Voronoi regions traversed by walking from u to v along the segment uv. See Figure 1 (a) for an illustration. If a Voronoi edge or a Voronoi vertex happens to lie on the segment uv, then choose the Voronoi region lying above uv. Assume that the line uv is the x-axis. The sequence of nodes b_i, $0 \leq i \leq m$, defines a path from u to v. In general, they [18] refer to the path constructed this way between some nodes u and v as the *direct DT path* from u to v. If the direct DT path connecting u and v is lying entirely above or entirely below the segment uv, it is called *one-sided*.



(a) Direct Delaunay path (b) Next neighbor

Fig. 1. (a) The direct DT path $ub_1b_2b_3b_4v$ between u and v shown by thickest lines. The tunnel $T(u, v)$ is shown by shaded lines. The thin lines represent the Voronoi diagram. (b) Find the next neighbor of node b_i in the direct DT path or the neighbor of x in the shortcut path.

Define the *tunnel*, denoted by $T(u, v)$, of segment uv as the set of triangles in the Delaunay triangulation, whose interior intersects the segment uv. The triangles illustrated in Figure 1 (a) is the tunnel $T(u, v)$ defined for nodes u and v.

The path constructed by Dobkin *et al.* uses the direct DT path as long as it is above the x-axis. Assume that the path constructed so far has brought us to some node b_i such that $y(b_i) \geq 0$, $b_i \neq v$, and $y(b_{i+1}) < 0$. Let j be the least integer larger than i such that $y(b_j) \geq 0$. Notice that here j exists because $y(b_m) = 0$ by assuming that uv is the x-axis. Then the path constructed by Dobkin *et al.* uses either the direct DT path from b_i to b_j or takes a *shortcut*, which is the upper boundary of the tunnel $T(u, v)$ that connects b_i and b_j. See [18] for more detail about the condition when to choose the direct DT path from b_i to b_j and when to choose the shortcut path from b_i to b_j. Let x_i, x_j be the x-coordinates of b_i and b_j respectively. Let $c_{dfs} = (1 + \sqrt{5})\pi/2$. It was proved in [18] that either the length of the direct DT path from b_i to b_j is at most $c_{dfs}(x_j - x_i)$ or the length of the shortcut between b_i and b_j is at most $c_{dfs}(x_j - x_i)$. For example, in Figure 1 (a), node b_2 is below the axis uv. Thus, node u either takes path ub_2b_3 or path uxb_3 to node b_3. Path ub_2b_3 is the direct DT path, which is below the axis. Path uxb_3 is the shortcut path from u to b_3.

Routing the packets along the direct DT path is a localized routing method, but it is not competitive on its own for all Delaunay triangulations. Bose and Morin [2] presented an example such that the distance traveled in this approach could be arbitrarily larger than the minimum. The routing strategy by Bose and Morin uses the direct DT path as long as it is above the x-axis. When the direct DT path lead us to an edge b_ib_{i+1} that intersects uv, it either continues to use the direct DT path or the shortcut to node b_j. The difficulty occurs as the strategy does not know prior which of these two paths is shorter. Their solution is to simulate exploring both paths in a parallel manner whenever the first one reaches node b_j. However, many technique details need to be filled so it can be implemented. Basically, we have to answer the following questions: (1) how to find the neighbor in the direct DT path locally, (2) how to find the neighbor in the shortcut path locally, and (3) how to determine whether node b_j is reached. We call this routing method Delaunay triangulation based routing, denoted by DTR.

For simplicity, let $v_1 = u, v_2, \cdots, v_{k-1}, v_k = v$ be the k vertices of all b_i's that is on or above the segment uv.

Firstly, we study how to find the neighbor of a node b_i in the direct DT path locally. Since the Voronoi region of a vertex is always a convex region, line segment uv only intersects at most *two* Voronoi edges of a Voronoi region. In other words, the direct DT path is uniquely and well defined. Assume that the current vertex b_i wants to find its next neighbor in the direct DT path. Then node b_i can compute $Vor(b_i)$ locally since it knows all Delaunay edges incident on b_i and the Voronoi diagram is a dual of the Delaunay triangulation. Then node b_{i+1} is the node that (1) shares the Voronoi edge of $Vor(b_i)$ that is intersected by uv, and (2) has larger x-coordinate than node b_i. See Figure 1 (b).

Secondly, we show how to find the next neighbor of a node x in the shortcut path locally. Remember that the shortcut path is the boundary segments of $T(u, v)$, which connects two consecutive vertices v_i and v_{i+1}, of tunnel $T(u, v)$. Vertex x first sorts all Delaunay edges incident on x in count-clockwise order. Then x finds the incident neighbor vertex w such that xw does not intersect the segment uv, but the previous Delaunay edge intersects uv. See Figure 1 (b) for an illustration. Here node w is the next node on the short-cut path.

Thirdly, we reach the node b_j if the following conditions hold: (1) the Voronoi diagram of the current node intersects uv, (2) the y-coordinate is not negative, and (3) if we are exploring the shortcut path, then the Voronoi Diagram of the previous visited node does not intersect uv; if we are exploring the direct shortcut path, then the y-coordinate of the previous visited node is negative. In Figure 1 (b), node w will be that node b_j.

The routing algorithm works as following. Let $v_0 = u$ and $i = 0$. Let node v_{i+1} be the node returned by EXPLORE(v_i). If v_{i+1} is not node v, then increase i by one and continue EXPLORE(v_i). The following is the detailed description of the algorithm EXPLORE(v_i).

Algorithm 1: EXPLORE(v_i)
Let p_0 be the next neighbor of v_i in the direct DT path, and q_0 be the next neighbor of v_i in the shortcut path. Let $j = 0$ and $l_0 = \min(\|v_ip_0\|, \|v_iq_0\|)$. Repeat the following exploring until a node, which is on the direct DT path and is above the segment uv, is reached. We denote such node by v_{i+1}. If $\|v_ip_0\| \leq \|v_iq_0\|$, we explore the direct DT path first. Otherwise, we have to explore the shortcut path first.

1) EXPLORE DIRECT DT PATH: Route the packet along the direct DT path from node v_i until reaching node v_{i+1} or reaching a node, say p_{j+1}, such that the distance traveled from p_0 to p_{j+1} is larger than $2l_j$ for the first time.
   If node v_{i+1} is reached, return v_{i+1} and quit. Otherwise, set $j = j + 1$ and l_j be the distance traveled from p_0 to p_{j+1}, and then return to node v_i.
2) EXPLORE SHORTCUT PATH: Route the packet along the shortcut path from node v_i until reaching node v_{i+1} or reaching a node, say q_{j+1}, such that the distance traveled from q_0 to q_{j+1} is larger than $2l_j$ for the first time.
   If node v_{i+1} is reached, return v_{i+1} and quit. Otherwise, set $j = j + 1$ and l_j be the distance traveled from q_0 to q_{j+1}, and then return to node v_i.

Notice that, originally, Bose and Morin [2] always start exploring the shortcut path first. However, this may lead to a long traveling distance when the first edge of the shortcut path is much longer than the direct DT path. Morin [23] proved the following theorem.

Theorem 1: The distance traveled by the above routing strategy is $9c_{dfs}$-competitive.
PROOF. Assume that the EXPLORE algorithm starts from node b_i and ends with node b_j. It was proved in [18] that either the length of the direct DT path from b_i to b_j is at most $c_{dfs}(x_j - x_i)$ or the length of the shortcut between b_i and b_j is at most $c_{dfs}(x_j - x_i)$. We only have to show that the actual distance traveled by the EXPLORE algorithm is at most9 times the distance between b_i and b_j, denoted by L. Notice that, $l_j \leq 2^j l_0$ and the distance from p_0 to p_j is traveled back and forth. The total distance traveled by exploring the direct DT path is at most $\sum_{j=0}^{k} 2l_j \leq \sum_{j=0}^{k} 2 \cdot 2^j l_0 \leq 4L$, where k is the maximum integer such that $l_k < L$. Similarly, the total distance traveled by exploring the shortcut path is at most $4L$. At last, it travels distance L when node b_j is reached. Thus, total traveled distance is at most $9L$. The theorem follows from $L \leq c_{dfs}(x_j - x_i)$. □

B. Construct Delaunay Locally

Although the above method works perfectly if the Delaunay triangulation of the set of nodes is known in advance, it is communication-intensive to construct the Delaunay triangulation in a distributed manner in the worst case. We will show that the Delaunay triangulation can be constructed using some localized approach with high probability when the nodes are randomly distributed and the transmission range is larger than some threshold (with high probability we can do so when the network is connected). Gupta and Kumar [16] showed that the unit disk graph is connected with high probability if the transmission range r_n satisfies $\pi \cdot r_n^2 \geq \frac{\ln n + c(n)}{n}$ for any $c(n)$ with $c(n) \to \infty$ as n goes infinity. Our construction is based on the local Delaunay triangulation by showing that all edges in the Delaunay triangulation is no more than the transmission radius with high probability when the nodes are randomly and uniformly distributed.

We assume that the wireless nodes are randomly and uniformly distributed in a unit area disk. It was proved in several papers [16], [24] that the random point process bears the same stochastic property as the homogeneous Poisson point process. The standard probabilistic model of *homogeneous Poisson process* is characterized by the property that the number of nodes in a region is a random variable depending only on the area (or volume in higher dimensions) of the region and the density of the process. Let λ be the density. The probability that there are exactly k nodes appearing in any region Ψ of area A is $\frac{(\lambda A)^k}{k!} \cdot e^{-\lambda A}$. Here after, we let $\mathcal{P}_n$ be a homogeneous Poisson process of intensity n on the unit area disk. We will consider the homogeneous Poisson point process instead of the random point process in our proof.

Let D be the variable denoting the length of the longest edge pq of the Delaunay triangulation of all wireless nodes generated by a homogeneous Poisson process with density n. Consider any edge e with length ℓ contained in some triangle $\triangle pqs$. Then the circumcircle of triangle $\triangle pqs$ has area at least $\pi\ell^2/4$. This circumcircle must contain no other nodes inside from the property of the Delaunay triangulation. The probability, denoted by p_1, that this circumcircle is empty of nodes is $\frac{(n\pi\ell^2/4)^0}{0!} \cdot e^{-n\pi\ell^2/4} = e^{-n\pi\ell^2/4}$. The probability that the longest edge of the Delaunay triangulation T is d_n satisfies

$$Pr(D \geq d_n) = Pr(\cup_{e \in T} e \geq d_n)$$
$$\leq \sum_{e \in T} Pr(e \geq d_n) \leq 3n \cdot e^{-n\pi d_n^2/4}$$

Notice that, there are at most $3n$ edges in the Delaunay triangulation of n two-dimensional nodes. By solving the inequality $3n \cdot e^{-n\pi d_n^2/4} \leq \frac{1}{\beta}$, we know that, with probability at most $\frac{1}{\beta}$, the longest edge of the Delaunay triangulation has length d_n, where $\pi d_n^2 \geq 4\frac{\ln n + \ln \beta + \ln 3}{n}$. In other words, with probability at least $1 - \frac{1}{\beta}$, the longest edge of the Delaunay triangulation has length d_n, where

$$\pi d_n^2 \leq 4\frac{\ln n + \ln \beta + \ln 3}{n}.$$

Penrose [25] showed that the longest edge of the minimum spanning tree of homogeneous Poisson point process $\mathcal{P}_n$ is at most M_n with probability $e^{-e^{-\alpha}}$, where $n\pi M_n^2 \leq \ln n + \alpha$. In other words, if the transmission radius r_n satisfies

$$\pi r_n^2 \geq \frac{\ln n + \alpha}{n},$$

then the induced graph $G(V, r_n)$ is connected with probability at least $e^{-e^{-\alpha}}$ when n goes infinity. By substituting $e^{\alpha} = \gamma$, we know that, with probability at least $1 - \frac{1}{\gamma}$, the induced unit disk graph is connected if the transmission range r_n of every node satisfies that

$$\pi r_n^2 \geq \frac{\ln n + \ln \gamma}{n}.$$

Combining the above analysis, the induced network is a connected graph with probability at least $1 - \frac{1}{n^7}$ if $\pi r_n^2 \geq \frac{8 \ln n}{n}$; meanwhile, with probability at least $1 - \frac{1}{n}$, the longest edge d_n of the Delaunay triangulation is at most r_n. Note that to make the induced network connected with probability $1 - \frac{1}{n}$, we need set the transmission radius r_n satisfies $\pi r_n^2 \geq \frac{2 \ln n}{n}$. In other words, the required transmission range so that local Delaunay triangulation equals the Delaunay triangulation is just twice of the minimum transmission range to have a connected network with high probability. Practically, the transmission range is often larger than the minimum requirement to get connectivity with high probability.

In the previous analysis, we did not consider the boundary effects. Our simulation results will show that the Delaunay edges near the domain boundary is often larger than the expected value of theoretical analysis for the domain without boundary. This is due to two reasons. First, our theoretical analysis holds only when n is large enough. Second, when the geometry domain in which the wireless nodes are distributed is bounded, the circumcircle of the Delaunay triangle near the domain boundary is not fully contained in the geometry domain. Thus, the probability that the circumcircle is empty of other nodes does not depend on the area of the circumcircle; instead, it depends on the area of the intersection of the circumcircle with the geometry domain. Our theoretical analysis for this boundary effect is omitted due to space limit.

C. Local Delaunay Triangulation

Since constructing Delaunay triangulation in a distributed manner is communication-intensive, we will rely on some localized construction method, more specifically, localized Delaunay triangulation [1]. For completeness of presentation, we give a brief review of the definition of the local Delaunay triangulation.

A triangle $\triangle uvw$ satisfies *k-localized Delaunay property* if (1) the interior of $disk(u, v, w)$ does not contain any node of V that is a k-neighbor of u, v, or w; (2) all edges of the triangle $\triangle uvw$ have length no more than one unit. Triangle $\triangle uvw$ is called a *k-localized Delaunay triangle*. The *k-localized Delaunay graph* over a node set V, denoted by $LDel^{(k)}(V)$, has exactly Gabriel edges and edges of k-localized Delaunay triangles. When it is clear from the context, we will omit the integer k in our notation of $LDel^{(k)}(V)$. Li *et al.* [1] proved that $LDel^{(k)}(V)$ is a planar graph for $k \geq 2$, but $LDel^{(1)}(V)$ may have intersecting edges.

Let $UDel(V)$, the *unit Delaunay triangulation*, be the graph obtained by removing all edges of $Del(V)$ that are longer than one unit. They proved that $UDel(V)$ is a spanner for UDG and is a subgraph of the k-localized Delaunay graph $LDel^{(k)}(V)$. They presented a localized method to extract from $LDel^{(1)}(V)$ a planar graph $PLDel$ containing $UDel(V)$ using only $O(n)$ communications total. See [1] for detail.. Thus, $PLDel$ is a t-spanner of the unit-disk graph $UDG(V)$. If the longest edge of the Delaunay triangulation is at most one unit, obviously, $PLDel$ is the Delaunay triangulation actually.

IV. Experiments

We first study the transition phenomena of the longest edge of the Delaunay triangulation. In our experiments, three different geometry regions Ω: disk with radius $200m$, square with side $400m$, and unbounded region of grids (with unit $400m$), are tested. The node density n is 50, 100, 200, 300, 400, and 500. For each choice of Ω and n, 10000 sample of n points is generated, and the longest Delaunay edge is generated for each sample. Left figures of Figure 2 illustrate the longest Delaunay edge length D_n distribution, while the right figures illustrate its transition phenomena. The statistics for D_n is from 0 to 400 meters, using 4 meters increment. Interestingly, for square

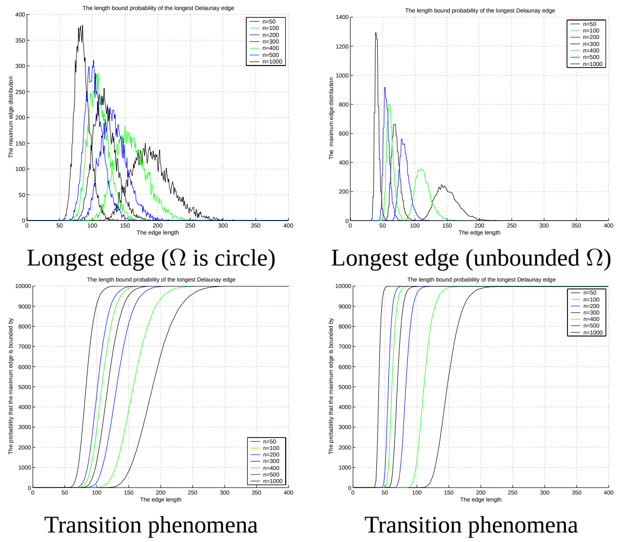


Fig. 2. Transition phenomena of D_n when Ω is circle, square, and unbounded.

region, varying density n does not change the distribution and transition at all statistically. The transition in the circular region is slower than the counterpart in the unbounded region. We found $D_n \leq 130m$ almost surely for circular region with $n = 100$.

We then present our experiments of various routing methods on different topologies. We choose 100 nodes distributed randomly in a circular area with radius 100 meters. Each node is specified by a random x, y coordinate, with transmission radius 30 meters. Figure 3 illustrates some discussed topologies. We randomly select 20% of nodes as source; and for each source, we randomly choose 20% of nodes as destination. The statistics are computed over 10 different node sets. We found that $LDel^{(2)}(V)$ and $PLDel(V)$ are almost the same as $Del(V)$. The differences lye near the boundary. These two Graphs are preferred over the Yao graph because we can apply the right hand rule when the simple heuristic localized routing fails.

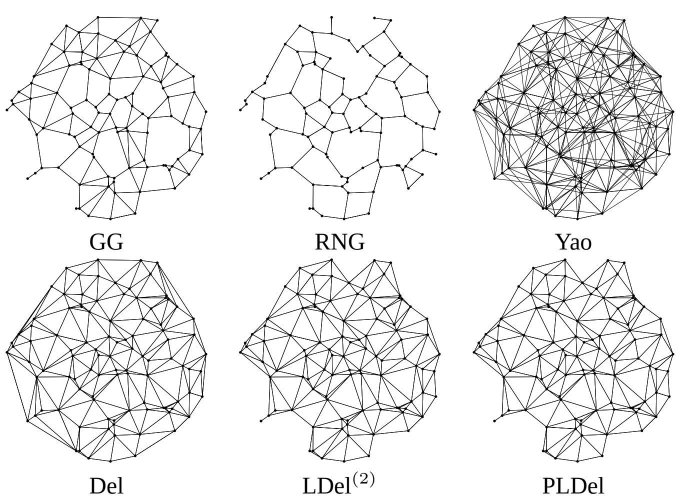


Fig. 3. Various planar network topologies (except Yao).

Interestingly, we found that when the underlying network topology is Yao graph, $Del(V)$, $LDel^{(2)}(V)$, or $PLDel(V)$, the compass routing, random compass routing and the greedy routing delivered the packets in all our experiments. Notice that it was proved that the Delaunay triangulation guarantees the delivery of the packets for these three routing methods. We also found that the local Delaunay triangulation and the planarized local Delaunay triangulation are almost the same as the Delaunay triangulation. The only differences lye near the domain boundary, which does not affect the localized routing too much. Thus, as we expected, the compass routing, random compass routing and the greedy routing delivered the packets in all our simulations for Delaunay related structures. The reason they also delivered the packets when Yao structure is used as the underlying topology could be there is a node within the transmission range in the direction of the destination with high probability when the number of nodes within transmission range is large enough.

TABLE I

THE DELIVERY RATE.

	Yao	RNG	GG	Del	LDel$^{(2)}$	PLDel
NN	98.7	44.9	83.2	99.1	97.8	98.3
FN	97.5	49	81.7	92.1	97	97.6
MFR	98.5	78.5	96.6	95.2	96.6	99.7
Cmp	100	86.6	99.6	100	100	100
RCmp	100	91.7	99.9	100	100	100
Grdy	100	87.5	99.6	100	100	100
GCmp	93	95.5	99.9	100	100	100
DTR				100	100	100

Table I illustrates the delivery rates of different localized routing protocols on various network topologies. For nearest neighbor routing and farthest neighbor routing, we choose the angle $\alpha = \pi/3$. In other words, we only choose the nearest node or the farthest node within $\pi/3$ of the destination direction. The $LDel^{(2)}(V)$ and $PLDel(V)$ graphs are preferred over the Yao graph because we can apply the right hand rule when previous simple heuristic localized routing fails. Both [3] and [10] use the greedy routing on Gabriel graph and use the right hand rule when greedy fails.

TABLE II

THE MAXIMUM SPANNING RATIO.

	Yao	RNG	GG	Del	LDel$^{(2)}$	PLDel
NN	1.9	2.1	1.9	1.7	1.8	1.9
FN	4.2	2.8	2.7	5.2	3.4	3.1
MFR	4.8	3.2	2.4	4.5	3.9	4.1
Cmp	3.3	2.9	2.8	1.6	1.8	2.0
RCmp	2.7	3.0	2.4	1.7	2.0	1.8
Grdy	2.1	3.5	2.2	2.0	1.9	1.9
GCmp	2.8	3.2	2.6	1.7	1.8	2.0
DTR				6.4	6.4	6.5

Table II illustrates the maximum spanning ratios of $\|\Pi(s,t)\|/\|st\|$, where $\Pi(s,t)$ is the path traversed by the packet using different localized routing protocols on various network topologies from source s to destination t. Because the localized Delaunay triangulation is much dense than all previous known planar network topologies such as Gabriel graph and the relative neighborhood graph, the delivery rates of many online routing methods are near or equal 100%. However, we have to admit that the traveled distance by the Delaunay based routing method DTR is larger than that by most previous methods for most source and destination pairs, although the actual distance of the traveled path is at the same level. Remember that, Delaunay based routing method has to travel some path back and forth to explore a better path. Nevertheless, Delaunay based routing is the only method known that can guarantee that the total traveled distance by the packet is within a constant factor of the minimum in any case.

We also conducted extensive simulations of the Face routing method on Gabriel graph and the local Delaunay triangulation $LDel^1(V)$. We choose $n = 20, 30, \cdots, 90, 100$ nodes randomly and uniformly distributed in a square of length 100 meters. The uniform transmission range of nodes are set as r, where r varies from $30, 40, 50, 60, 70$ meters. Table III illustrates the averaged spanning ratio achieved. The average is computed for all pair of nodes. Given n and r, we generate 10 sets of random n points. We found that the spanning ratio of the Face routing method is significantly less when local Delaunay triangulation is used instead of Gabriel graph. It may be due to local Delaunay triangulation has more edges, thus the faces traversed by the Face routing algorithm is often smaller when LDel is used than the case when GG is used.

TABLE III

THE AVERAGE SPANNING RATIO OF FACE ROUTING METHODS ON GABRIEL GRAPH AND LOCAL DELAUNAY TRIANGULATION.

n	0.3	0.4	0.5	0.6	0.7
20	3.2	3.0	2.9	2.9	2.9
30	4.7	4.8	4.6	4.4	4.3
40	5.0	5.2	5.1	5.0	5.1
50	5.5	5.9	5.9	5.7	5.3
60	6.1	6.1	6.1	6.3	6.0
70	6.5	6.5	6.6	6.4	6.6
80	6.9	6.7	7.1	6.9	6.6
90	7.0	7.1	7.4	7.5	7.1
100	7.3	7.4	7.7	7.3	7.3

On Gabriel graph

n	0.3	0.4	0.5	0.6	0.7
20	2.9	2.8	2.9	2.7	2.7
30	4.4	4.5	4.4	4.5	4.1
40	5.2	5.5	4.8	4.8	4.9
50	4.9	5.3	5.4	5.5	5.3
60	5.3	5.7	5.4	5.7	6.1
70	5.9	5.6	6.1	6.2	5.8
80	5.9	6.1	6.4	5.9	5.8
90	6.0	6.4	6.5	6.4	6.0
100	6.4	6.5	6.8	6.6	6.5

On local Delaunay triangulation

V. CONCLUSION

In this paper, we showed that, given a set of randomly distributed wireless nodes over a region with node density n, when the transmission range r_n satisfies $\pi r_n^2 \geq \frac{8 \log n}{n}$, the localized Delaunay triangulation equals the Delaunay triangulation with probability almost $1-\frac{1}{n}$. If $\pi r_n^2 \geq \frac{8 \log n}{n}$, the induced network topology is connected with probability at least $1 - \frac{1}{n^7}$. In other words, with high probability, we can construct the Delaunay triangulation using the localized Delaunay triangulation if the network is connected. Thus, we can apply a localized routing protocol [2] that guarantees that the distance traveled by the packets is no more than a small constant factor of the minimum. We also conducted experiments to show that the delivery rates of existing localized routing protocols are increased when localized Delaunay triangulation is used instead of several previously proposed topologies. Notice that the Delaunay based routing method DTR works only when a Delaunay triangulation is obtained. Currently, when we found that Delaunay triangulation is not constructed, we rely on other heuristic to route the packets. We leave it as a future work to design a protocol that can guarantee the traveled distance using only the localized Delaunay triangulation.

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