Capacity Scaling Laws of Cognitive Networks

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Abstract

In this paper, we study the capacity of *cognitive networks*. We focus on the network model consisting of two overlapping ad hoc networks, called the primary ad hoc network (PaN) and secondary ad hoc network (SaN), respectively. PaN and SaN operate on the same space and spectrum. For PaN (or SaN resp.) we assume that primary (or secondary resp.) nodes are placed according to a Poisson point process of intensity n (or m resp.) over a unit square region. We randomly choose n_s (or m_s resp.) nodes as the sources of multicast sessions in PaN (or SaN resp.), and for each primary source v^p (or secondary source v^s), we pick uniformly at random n_d primary nodes (or m_d secondary nodes) as the destinations of v^p (or v^s). Above all, we assume that PaN can adopt the optimal protocol in terms of the throughput. Our main work is to design the multicast strategy for SaN by which it can achieve the optimal throughput, without any negative impact on the throughput for PaN in order sense.

Depending on n_d and n, we choose the optimal strategy for PaN from two ones called *percolation strategy* and *connectivity strategy*, respectively. Subsequently, we design the corresponding throughput-optimal strategy for SaN. We further derive the regimes for n, n_d, m and m_d where the throughput for PaN and SaN can simultaneously be achieved of the upper bound of their capacities asymptotically. Specifically, we show that (1) when $n = o(\frac{m}{(\log m)^2})$, $n_d = O(\frac{n}{(\log n)^3})$, and $m_d = O(\frac{m}{(\log m)^3})$, PaN and SaN can simultaneously achieve the optimal throughput, *i.e.*, the upper bounds of the capacity ($\Theta(\sqrt{n \cdot n_d})$ and $\Theta(\sqrt{m \cdot m_d})$, respectively); (2) when $n_d = \Omega(\frac{n}{\log n})$ and $m_d = \Omega(\frac{m}{\log m})$, PaN and SaN also simultaneously achieve the optimal throughput, *i.e.*, $\Theta(1)$ for both networks. *Unicast capacity* (or *broadcast capacity*) for the cognitive network can be derived by our results as special cases by letting $n_d = 1$ and $m_d = 1$ (or $n_d = n - 1$ and $m_d = m - 1$).

Index Terms

Cognitive networks, wireless ad hoc networks, multicast capacity, random networks, percolation theory.

I. Introduction

The demand for bandwidth is increasing. However, a large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum ranges from 15% to 85% with a high variance in time [1], [2]. A solution to the issue that the limited available spectrum co-exists with the inefficiency in the spectrum usage is to permit some users to exploit the wireless spectrum opportunistically without having a negative impact on the licensed users. Thus, a new communication paradigm, *i.e.*, *cognitive network*, was proposed. A cognitive network generally consists of two independent overlapping networks, called the *primary networks* and *secondary networks*, that operate at the same time, space and frequency. The secondary users are equipped with cognitive radios and are able to sense the idle spectrum and obtain the necessary information of primary users [3], [4].

In this paper, we study the capacity of networks consisting of the primary ad hoc network (PaN) and the secondary ad hoc network (SaN). We assume that the nodes of PaN and SaN are distributed according to a Poisson point process (p.p.p.) of intensity n and m respectively over a square. We will focus on the multicast capacity that unifies results on unicast and broadcast capacity, [5], [6], [12]. Due to the nature of wireless medium, both the primary network and secondary network have impact on each other under the noncooperative communication scenario as long as they share the same spectrum in the same time. We note that the communications of secondary users should be non-destructive to the communications of primary users. Thus, the upper bounds on the capacity for PaN is no more than that for the single network isomorphic to PaN. Similarly, it holds for SaN.

The most important constraint for cognitive networks is that the primary network does not alter its protocol due to the secondary network anyway, [7]. Otherwise, a simple equal time-sharing can achieve the same order of throughput for both networks as they are stand-alone, which makes the problem trivial. Under the constraint, a challenging issue is whether there exist communication strategies for PaN and SaN by which they can simultaneously achieve the upper bounds on capacity of their isomorphic single ad hoc networks. We answer the question positively by proposing two types of multicast strategies. The first one is *percolation strategy* in which the routing scheme is constructed based on the *highways* system ([8], [9]). The second one is *connectivity strategy* in which the routing is based on *connectivity paths*, in which the link length is in the order of the minimum for ensuring global network connectivity. A key technique is to set the *preservation region* for each primary user. Therefore, the difference between the strategies for SaN and that for PaN is that the *preservation regions* can not be passed through by any transmission in SaN. For both networks, there are the thresholds on the number of destinations for each multicast session, below which *percolation strategies* do perform better than the others. Based on those thresholds, we use the two types of strategies accordingly, and derive their corresponding throughputs. We show that if SaN is denser than PaN, for some cases, PaN and SaN can simultaneously achieve the upper bounds of their respective capacities.

The rest of the paper is organized as follows. In Section II we introduce the system model. In Section III, main results are presented. We make technical preparations in Section IV. In Section V we propose the multicast strategies. In Section VI we derive the achievable throughput and prove main results. We review the related work in Section VII. We conclude the paper in Section VIII.

II. System Model

Throughout the paper, we mainly consider the event that happens with high probability (*w.h.p.*) as the scale of network (the number of users in the network) goes to infinity.

NOTATIONS: In the paper, we adopt the following notations:

- $x \to \infty$ denotes that variable x takes value to infinity.
- For a discrete set $\mathcal{U}$, $|\mathcal{U}|$ represents its cardinality.
- For a continuous region $\mathcal{A}$, let $\|\mathcal{A}\|$ denote its area.
- For a 2-dimension line segment $\mathcal{L} = uv$, $\|\mathcal{L}\|$ represents its Euclidean length. For a tree $\mathcal{T}$, denote its total Euclidean edge lengths by $\|\mathcal{T}\|$.
- For Event E, denote the probability of E as $\Pr(E)$.
- The notion $\theta(n)\sim[\theta_1(n), \theta_2(n)]$ represents that
$$\theta(n) = \Omega(\theta_1(n)) \text{ and } \theta(n) = O(\theta_2(n)),$$
while $\theta(n)\sim(\theta_1(n), \theta_2(n)]$ means that
$$\theta(n) = \omega(\theta_1(n)) \text{ and } \theta(n) = O(\theta_2(n)).$$

A. Network Topology

Denote PaN and SaN respectively by

$$\mathcal{N}_p(n) = (\mathcal{V}_p(n), \mathcal{E}_p(n)) \text{ and } \mathcal{N}_s(m) = (\mathcal{V}_s(m), \mathcal{E}_s(m))$$

where $\mathcal{V}_p(n)$ (or $\mathcal{V}_s(m)$ resp.) and $\mathcal{E}_p(n)$ (or $\mathcal{E}_s(m)$ resp.) are the set of nodes and edges of $\mathcal{N}_p(n)$ (or $\mathcal{N}_s(m)$ resp.). The nodes of PaN and SaN are distributed according to a Poisson point processes (p.p.p.) of intensity n and m respectively over a unit square $\mathcal{A} = [0, 1]^2$, i.e., we consider the *dense network model* [8], [10]. From Chebychev's inequality, we can easily obtain, *w.h.p.*, the number of primary nodes (or secondary nodes) , *i.e.*, $|\mathcal{V}_p(n)|$ (or $|\mathcal{V}_s(m)|$ resp.), is within $[(1-\varepsilon)n, (1+\varepsilon)n]$ (or $[(1-\varepsilon)m, (1+\varepsilon)m]$ resp.). To simplify description, assume that $|\mathcal{V}_p(n)| = n$ and $|\mathcal{V}_s(m)| = m$ respectively as in [8], [11], which does not change our asymptotic results.

B. Gaussian Channel Model

Both networks are assumed to operate based on TDMA scheme. The time slots of two networks are assumed to have equal length, however, the scheduling periods are unnecessarily equal. Let $\mathcal{V}(\tau)$ denote the set of nodes scheduled at slot τ. Then, during any time slot τ, a node $v_i \in \mathcal{V}(\tau)$ can send data to a node v_j via a direct link, over a channel with bandwidth B, of rate $\mathrm{R}(v_i, v_j) = B \log(1 + \frac{\mathrm{S}(v_i,v_j)}{\mathrm{N}_0+\mathrm{I}(v_i,v_j)})$, where N_0 is the ambient noise, $\mathrm{S}(v_i, v_j)$ is the strength of the signal initiated by v_i at the receiver v_j, $\mathrm{I}(v_i, v_j)$ is the sum interference on v_j produced by all nodes belong to $\mathcal{V}(\tau) - \{v_i\}$. Since no inter-communication occurs between two networks, we have

$$\mathrm{I}(v_i, v_j) = \begin{cases} \mathrm{I}_{pp}(v_i, v_j) + \mathrm{I}_{sp}(v_i, v_j), & \text{when } v_i, v_j \in \mathcal{V}_p(n) \\ \mathrm{I}_{ps}(v_i, v_j) + \mathrm{I}_{ss}(v_i, v_j), & \text{when } v_i, v_j \in \mathcal{V}_s(m) \end{cases}$$

where $\mathrm{I}_{pp}(v_i, v_j)$, or $\mathrm{I}_{ps}(v_i, v_j)$, denotes the sum of interference on v_j produced by all nodes in $\mathcal{V}_p(v_i, \tau) = \mathcal{V}(\tau) \bigcap \mathcal{V}_p(n) - \{v_i\}$; $\mathrm{I}_{sp}(v_i, v_j)$, or $\mathrm{I}_{ss}(v_i, v_j)$, denotes the sum of interference on v_j produced by all nodes in $\mathcal{V}_s(v_i, \tau) = \mathcal{V}(\tau) \bigcap \mathcal{V}_s(m) - \{v_i\}$.

The wireless propagation channel typically includes path loss with distance, shadowing and fading effects. In this paper, as in [8], [10], we assume the channel gain depends only on the distance between a transmitter and receiver, and ignore shadowing and fading. The channel power gain $\ell(v_i, v_j)$ is given by $\ell(v_i, v_j) = (d(v_i, v_j))^{-\alpha}$, where $d(v_i, v_j) = \|v_i v_j\|$ is the Euclidean distance between two nodes v_i and v_j, $\alpha > 2$ is the power attenuation exponent. Notice that our results hold as long as *near field effects of electromagnetic propagation* can be neglected.

For $\mathcal{N}_p(n)$, at the time slot τ, any pair $v_i^p \in \mathcal{V}_p(n) \bigcap \mathcal{V}(\tau)$ and $v_j^p \in \mathcal{V}_p(n)$ can achieve the rate of

$$\mathrm{R}_p(v_i^p, v_j^p) = B \log \left(1 + \frac{\mathrm{S}(v_i^p, v_j^p)}{N_0 + \mathrm{I}_{pp}(v_i^p, v_j^p) + \mathrm{I}_{sp}(v_i^p, v_j^p)} \right)$$

where

- $\mathrm{S}(v_i^p, v_j^p) = P(v_i^p) \cdot \frac{1}{\|v_i^p v_j^p\|^\alpha}$.
- $\mathrm{I}_{pp}(v_i^p, v_j^p) = \sum_{v_k^p \in \mathcal{V}_p(v_i^p, \tau)} P(v_k) \cdot \frac{1}{\|v_k^p v_j^p\|^\alpha}$.
- $\mathrm{I}_{sp}(v_i^p, v_j^p) = \sum_{v_k^s \in \mathcal{V}_s(v_i^p, \tau)} P(v_k) \cdot \frac{1}{\|v_k^s v_j^p\|^\alpha}$.

For $\mathcal{N}_s(m)$, at the time slot τ, any pair $v_i^s \in \mathcal{V}_s(m) \bigcap \mathcal{V}(\tau)$ and $v_j^s \in \mathcal{V}_s(m)$ can achieve the rate of

$$\mathrm{R}_s(v_i^s, v_j^s) = B \log \left(1 + \frac{\mathrm{S}(v_i^s, v_j^s)}{N_0 + \mathrm{I}_{ps}(v_i^s, v_j^s) + \mathrm{I}_{ss}(v_i^s, v_j^s)} \right)$$

where

- $\mathrm{S}(v_i^s, v_j^s) = P(v_i^s) \cdot \frac{1}{\|v_i^s v_j^s\|^\alpha}$.
- $\mathrm{I}_{ps}(v_i^s, v_j^s) = \sum_{v_k^p \in \mathcal{V}_p(v_i^s, \tau)} P(v_k) \cdot \frac{1}{\|v_k^p v_j^s\|^\alpha}$.
- $\mathrm{I}_{ss}(v_i^s, v_j^s) = \sum_{v_k^s \in \mathcal{V}_s(v_i^s, \tau)} P(v_k) \cdot \frac{1}{\|v_k^s v_j^s\|^\alpha}$.

C. Capacity Definition

We propose the formal definition of capacity based on that in [5], [12]. Let $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ denote the set of all nodes in the network and let the subset $\mathcal{S} \subseteq \mathcal{V}$ denote the set of source nodes of multicast. Let the number of multicast sessions be $|\mathcal{S}| = n_s$. For each source $v_{\mathcal{S},i} \in \mathcal{S}$, we uniformly choose n_d nodes at random from other nodes to construct $\mathcal{D}_{\mathcal{S},i} = \{v_{\mathcal{S},i_1}, v_{\mathcal{S},i_2}, \cdots, v_{\mathcal{S},i_{n_d}}\}$ as the set of destinations, where obviously $n_d \leq n-1$. We call $\mathcal{U}_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \cup \mathcal{D}_{\mathcal{S},i}$ the *spanning set* of multicast session $\mathcal{M}_{\mathcal{S},i}$. Denote $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ as a *rate vector* of the multicast data rate of all multicast sessions.

Definition 1 (Feasible Rate Vector): A multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ is called (ρ_s, ρ_d)-*feasible*, where ρ_s and ρ_d are both constants in $[0, 1]$, if for a subset of sources, denoted as $\mathcal{S}'(\rho_s, \rho_d) \subseteq \mathcal{S}$ satisfying $|\mathcal{S}'(\rho_s, \rho_d)| = \rho_s(n) \cdot n_s$, there exists a spatial and temporal scheme for scheduling transmissions by which every source $v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s, \rho_d)$ can deliver data to at least $\rho_d(n, i) \cdot n_d$ destinations at rate of $\lambda_{\mathcal{S},i}$. That is, there is a $T < \infty$ such that in every time interval (with unit seconds) $[(i-1) \cdot T, i \cdot T]$, every node $v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s, \rho_d)$ can send $T \cdot \lambda_{\mathcal{S},i}$ bits to at least its $\rho_d(n, i) \cdot n_d$ destinations, where

$$\lim_{n \to \infty} \rho_s(n) = \rho_s; \quad \lim_{n \to \infty} \inf_{v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s, \rho_d)} \{\rho_d(n, i)\} = \rho_d.$$

A multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ is called *feasible* if it is $(1, 1)$-*feasible*.

Based on a *multicast rate vector*, we can define the following three types of multicast throughput (MT).

- Aggregated multicast throughput (AMT)
  $\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n) = \sum_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$
- Average per-session multicast throughput (APMT)
  $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = \frac{1}{n_s} \sum_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$
- Minimum per-session multicast throughput (MPMT)
  $\Lambda^{\mathrm{M}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$

Correspondingly, we define three types of the *achievable* multicast throughput based on the *feasible rate vector*.

Definition 2: AMT $\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n)$ (or APMT $\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n)$, or MPMT $\Lambda^{\mathrm{M}}_{\mathcal{S},n_d}(n)$) is *achievable* if the multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \cdots, \lambda_{\mathcal{S},n_s})$ is *feasible*.

Definition 3 (Capacity of Random Networks): The aggregated multicast capacity of a class of random networks is of order $\Theta(g(n))$ bits/sec if there are deterministic constants $c > 0$ and $c < c' < +\infty$ such that

$$\lim_{n\to\infty} \Pr(\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n) = c \cdot g(n) \text{ is achievable}) = 1,$$
$$\liminf_{n\to\infty} \Pr(\Lambda^{\mathrm{T}}_{\mathcal{S},n_d}(n) = c' \cdot g(n) \text{ is achievable}) < 1.$$

We can similarly define the per-session and minimum per-session capacity for random networks. As in most related work, we assume that $n_s = \Theta(n)$.

III. Main Results

For both $\mathcal{N}_p(n)$ and $\mathcal{N}_s(m)$, we design two types of strategies called *percolation strategy* and *connectivity strategy*. In the following theorems, the results for PaN hold under the condition that $n = o(\frac{m}{\log m})$, and the results for SaN hold under Assumption A and Assumption B.

Assumption A: For PaN and SaN, we assume that

1) $\mathcal{N}_p(n)$ operates as $\mathcal{N}_s(m)$ were absent. That is, $\mathcal{N}_p(n)$ does not alter its protocol due to $\mathcal{N}_s(m)$ anyway.
2) Secondary nodes know the locations of primary nodes.

Assumption B: For m, m_d and n,

1) When $m_d = \omega(\log m)$, we assume that $n = o(\frac{m}{\log m})$.
2) When $m_d = O(\log m)$, we assume that $n = o(\frac{m}{m_d \cdot \log m})$.

Since the fact that APMT $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n)$ must be achievable if MPMT $\Lambda^{\mathrm{M}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$ is achievable. Then, we mainly focus on the following representative metrics.

- P-MPMT: The minimum per-session MT for PaN;
- S-MPMT: The minimum per-session MT for SaN;
- P-AMT: The aggregated MT for PaN;
- P-AMT: The aggregated MT for SaN;
- P-AMC: The aggregated multicast capacity for PaN;
- S-AMC: The aggregated multicast capacity for SaN.

Based on certain metrics above, other types of multicast throughput (or capacity) can be derived.

Before presenting the main results, we define two functions with positive integer domains as follows.

$$\mathrm{T}_1(x,y) = \begin{cases} \Omega(\frac{1}{\sqrt{xy}}) & \text{when } y \sim [1, \frac{x}{(\log x)^3}] \\ \Omega(\frac{1}{y} \cdot (\log x)^{\frac{3}{2}}) & \text{when } y \sim [\frac{x}{(\log x)^3}, x] \end{cases}$$
$$\mathrm{T}_2(x,y) = \begin{cases} \Omega(\frac{1}{\sqrt{xy \log x}}) & \text{when } y \sim [1, \frac{x}{\log x}] \\ \Omega(1/x) & \text{when } y \sim [\frac{x}{\log x}, x] \end{cases}$$

Theorem 1: When $n_d \sim [1, \frac{n}{(\log n)^2}]$ and $m_d \sim [1, \frac{m}{(\log m)^2}]$, the achievable throughput is of order as following.

- P-MPMT: $\Omega(\mathrm{T}_1(n, n_d))$, P-AMT: $\Omega(n \cdot \mathrm{T}_1(n, n_d))$
- S-MPMT: $\Omega(\mathrm{T}_1(m, m_d))$, S-AMT: $\Omega(m \cdot \mathrm{T}_1(m, m_d))$

Theorem 2: When $n_d \sim [1, \frac{n}{(\log n)^2}]$ and $m_d \sim [\frac{m}{(\log m)^2}, m]$, the achievable throughput is of order as following.

- P-MPMT: $\Omega(\mathrm{T}_1(n, n_d))$, P-AMT: $\Omega(n \cdot \mathrm{T}_1(n, n_d))$
- S-MPMT: $\Omega(\mathrm{T}_2(m, m_d))$, S-AMT: $\Omega(m \cdot \mathrm{T}_2(m, m_d))$

Theorem 3: When $n_d \sim [\frac{n}{(\log n)^2}, n]$, for all $m_d \sim [1,\ m]$,
the achievable throughput is of order as following.

- P-MPMT: $\Omega(\mathrm{T}_2(n, n_d))$, P-AMT: $\Omega(n \cdot \mathrm{T}_2(n, n_d))$
- S-MPMT: $\Omega(\mathrm{T}_2(m, m_d))$, S-AMT: $\Omega(m \cdot \mathrm{T}_2(m, m_d))$

Combining with the upper bound proposed in [6], [14] (described in Lemma 3), we obtain

Theorem 4: P-AMC is of order

$$\mathrm{C}_p^{\mathrm{A}}(n) = \begin{cases} \Theta(\frac{\sqrt{n}}{\sqrt{n_d}}) & \text{when } n_d \sim [1, \frac{n}{(\log n)^3}] \\ \Theta(1) & \text{when } n_d \sim [n/\log n, n] \end{cases}$$

And S-AMC is determined by PaN, *i.e.*,

When $n_d \sim [1, \frac{n}{(\log n)^2}]$, AMC for SaN is of order

$$\mathrm{C}_p^{\mathrm{A}}(m) = \begin{cases} \Theta(\frac{\sqrt{m}}{\sqrt{m_d}}) & \text{when } m_d \sim [1, m/(\log m)^3] \\ \Theta(1) & \text{when } m_d \sim [m/\log m, m] \end{cases}$$

When $n_d \sim [\frac{n}{(\log n)^2}, n]$, AMC for SaN is of order

$$\mathrm{C}_p^{\mathrm{A}}(m) = \Theta(1) \text{ when } m_d \sim [m/\log m, m]$$

According to Theorem 4, we get that, for the case that $n_d \sim [1, n/(\log n)^2]$ and $m_d \sim [m/(\log m)^3, m/\log m]$, and for the case that $n_d \sim [n/(\log n)^2, n]$ and $m_d \sim [1, m/\log m]$, there are still the gaps between the upper bounds and lower bounds. An interesting and challenging issue is to close those gaps by presenting possibly new tighter upper bounds, and lower bounds, and designing corresponding algorithms to achieve the asymptotic multicast capacity.

IV. Technical Preparations

First of all, we recall some useful results on bounds.

Lemma 1 (Mitzenmacher et al. [13]): Let X be a Poisson random variable with parameter λ. Then

$$\Pr(X \geq x) \leq e^{-\lambda}(e\lambda)^x/x^x, \quad \text{for } x > \lambda. \tag{1}$$

$$\Pr(X \leq x) \leq e^{-\lambda}(e\lambda)^x/x^x, \quad \text{for } 0 < x < \lambda. \tag{2}$$

Algorithm 1 Construction of EST

Input: The set of nodes $\mathcal{U}_{\mathcal{S},k}$

Output: $EST(\mathcal{U}_{\mathcal{S},k})$.

1: In the initial state, all nodes of $\mathcal{U}_{\mathcal{S},k}$ are isolated, then there are $n_d + 1$ connected components.

2: **for** $i = 1 : n_d$ **do**

3: Partition the deployment region $\mathcal{A} = [0,1]^2$ into at most $n_d + 1 - i$ square cells, each with side length $1/\lfloor\sqrt{n_d + 1 - i}\rfloor$;

4: Find a cell that contains two nodes of $\mathcal{U}_{\mathcal{S},k}$ that are from two different connected components. By connecting the pair of nodes, we merge the two connected components.

5: **end for**

Lemma 2 (Li et al. [5], [12]): For any $n_d + 1$ nodes U placed in a square with unit side-length, the length of EMST(U), obtained by Algorithm 1, is at most $2\sqrt{2} \cdot \sqrt{n_d}$.

Based on a technique called *arena* in [14], Keshavarz-Haddad *et al.* have derived the upper bound of the multicast capacity for *dense networks* in [6]. That is,

Lemma 3: The aggregated multicast capacity for the single *dense networks* isomorphic to PaN, is at most of order

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d \sim [1, \frac{n}{(\log n)^2}] \\ O(\frac{1}{n_d \cdot \log n}) & \text{when } n_d \sim [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ O(\frac{1}{n}) & \text{when } n_d \sim [\frac{n}{\log n}, n] \end{cases} \tag{3}$$

The analogue result for SaN holds by substituting m and m_d for n and n_d in Eq. (3).

Poisson Boolean Percolation Model: In 2-dimensional Poisson Boolean model $\mathcal{B}(\lambda, r)$ [15], nodes are distributed according to a p.p.p of intensity λ in $\mathbb{R}^2$. Each node is associated to a closed disk with radius $r/2$. Two disks are *directly connected* if they overlap. Two disks are *connected* if there exists a sequence of directly connected disks between them. Define a *cluster* as a set of disks in which any two disks are connected. Define the set of all clusters as $\mathcal{C}(\lambda, r)$. Define the number of disks in the cluster $C_i \in \mathcal{C}(\lambda, r)$ as a random variable $N(C_i)$. We can associate $\mathcal{B}(\lambda, r)$ to a graph $\mathcal{G}(\lambda, r)$, called *associated graph*, by associating a vertex to each node of $\mathcal{B}(\lambda, r)$ and an edge to each direct connection in $\mathcal{B}(\lambda, r)$. The two models $\mathcal{B}(\lambda, r)$ and $\mathcal{B}(\lambda_0, r_0)$ lead to the same associated graph, namely $\mathcal{G}(\lambda, r) = \mathcal{G}(\lambda_0, r_0)$ if $\lambda_0 {r_0}^2 = \lambda r^2$. Then, the graph properties of $\mathcal{B}(\lambda, r)$ depend only on the parameter λr^2, [10]. The *percolation probability*, denoted as $\mathfrak{p}$, is one that a given node belongs to a cluster with an infinite number of nodes. With C denoting the cluster containing the given node, the percolation probability is thus defined as $\mathfrak{p}(\lambda, r) = \mathfrak{p}(\lambda r^2) = \Pr_{\lambda,r}(|C| = \infty) = \Pr_{\mathfrak{p}}(|C| = \infty)$. We call $\mathfrak{p}_c$ the *critical percolation threshold* of Poisson Boolean model in $\mathbb{R}^2$ when $\mathfrak{p}_c = (\lambda r^2)_c = \sup\{\lambda r^2 | \mathfrak{p}(\lambda r^2) = 0\}$. The exact value of $(\lambda r^2)_c$ is not yet known. The best analytical results show that it is within $(0.7698,\ 3.372)$ [15], [16]. In our analysis, we will use the following lemma.

Lemma 4 (Meester and Roy [15]): For a Poisson Boolean model $\mathcal{B}(\lambda, r)$ in $\mathbb{R}^2$, if $\lambda r^2 < \mathfrak{p}_c$, it holds that

$$\Pr(\sup\{N(C_i) \mid C_i \in \mathcal{C}(\lambda, r)\} < \infty) = 1,$$

where $\mathfrak{p}_c$ is the *critical percolation threshold* of Poisson Boolean model in $\mathbb{R}^2$.

Bond Percolation Model: We mainly recall a result proposed in [8] that is to show the existence of a cluster of nodes forming the *highway system* ([8]). The result is derived based on the independent bond percolation model on the square lattice ([17]), where each edge (bond) of an infinite square grid is *open* with probability p and *closed* otherwise, independently of all other edges.

Let $\mathbb{B}(h, p)$ denote a box of side length h embedded in the square lattice. We call a path consisting of only open edges (bonds) *open path*. For a given $\kappa > 0$, we partition the lattice graph $\mathbb{B}(h, p)$ into horizontal (vertical) rectangle slabs with the horizontal (vertical) width of h and the vertical (horizontal) width of $\kappa \log h - \epsilon(h)$, denoted as R_i^h (R_i^v). Note that we can choose ε_h as the smallest value such that the number of rectangle slabs $h/(\kappa \log h - \epsilon(h))$ is an integer. It is obvious that $\epsilon(h) = o(1)$ as $h \to \infty$ [8]. Denote the number of edge-disjoint *open paths* in slab R_i^h (R_i^v) as N_i^h (N_i^v). Let $N^h = \min_i N_i^h$, $N^v = \min_i N_i^v$. Then, we have

Lemma 5: ([8]) For any constant $\kappa > 0$ and $p \in (\frac{5}{6}, 1)$ satisfying $2 + \kappa \log(6(1-p)) < 0$, there exists a constant $\delta(\kappa, p)$ depending on κ and p such that

$$\lim_{h\to\infty} \Pr(N^h \geq \delta \log h) = 1, \lim_{h\to\infty} \Pr(N^v \geq \delta \log h) = 1.$$

Hierarchical Routing: In general, the lower bounds on capacity can be obtained by designing the specific multicast strategy. We propose a class of multicast strategies, denoted as $\mathfrak{F}$, with routing scheme $\mathfrak{F}^r$ and transmission scheduling $\mathfrak{F}^t$. Notice that the routing scheme $\mathfrak{F}^r$ may have a hierarchical structure consisting of τ phases that correspond to *sub-routing schemes* $\mathfrak{F}^{r_1}, \mathfrak{F}^{r_2}, \cdots, \mathfrak{F}^{r_\tau}$, where $\tau \geq 1$ is a constant and it means that the routing scheme $\mathfrak{F}^r$ is non-hierarchical when $\tau = 1$. Let $\mathcal{V}(\mathfrak{F}^{r_j})$ represent the set of nodes that are passed through by some multicast sessions based on routing scheme $\mathfrak{F}^{r_j}$, where $j = 1, 2, \cdots, \tau$.

Definition 4 (Sufficient Region): For a node $v_i^j \in \mathcal{V}(\mathfrak{F}^{r_j})$, $1 \leq j \leq \tau$, and for any multicast session $\mathcal{M}_{\mathcal{S},k}$, we call a region $\mathcal{Q}(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ *sufficient region* if

$$\Pr(E(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)) \leq \Pr(\tilde{E}(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)) \tag{4}$$

where Event $E(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ is defined as: $\mathcal{M}_{\mathcal{S},k}$ is routed through v_i^j based on the sub-routing scheme $\mathfrak{F}^{r_j}$ during Phase j; Event $\tilde{E}(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ is defined as: A poisson node locates in the region $\mathcal{Q}(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$.

Lemma 6 (Throughput during Phase j): In the *dense network* deployed in a unit square $\mathcal{A} = [0, 1]^2$, if each node in $\mathcal{V}(\mathfrak{F}^{r_j})$ $(1 \leq j \leq \tau)$ can sustain a total rate of R_j by using the transmission scheduling $\mathfrak{F}^t$, and for $k = 1, 2, \cdots n_s$, there exists a value Q_j independent of k and i such that uniform *w.h.p.*, $\|\mathcal{Q}(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)\| \leq Q_j$. Then, when $Q_j = \Omega(\frac{\log n}{n})$, the achievable per-session throughput during Phase j is uniform *w.h.p.*, of order $\Lambda_j = \Omega(\frac{R_j}{n_s \cdot Q_j})$.

The proof is provided in Appendix.

Since the bottleneck determines the achievable throughput derived by the whole multicast strategy, we can clearly obtain the following proposition.

Proposition 1: The achievable throughput derived by multicast strategy $\mathfrak{F}$ is of $\Lambda = \min_j\{\Lambda_j\}$.

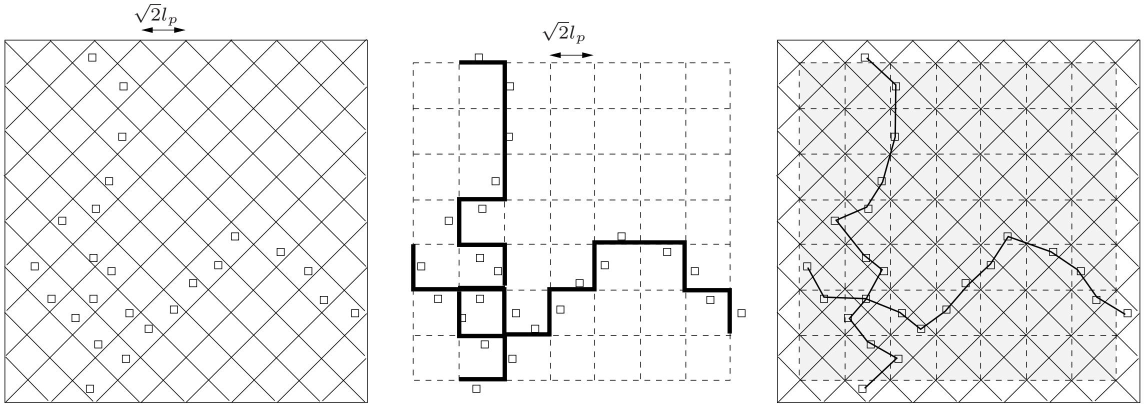


Fig. 1. Construction of highways. (a)Lattice $\mathbb{C}_p(h_p)$. An inclined cell is *open* if it contains at least one node. (b)Open Paths. Two *open paths* are depicted using bold square lines. (c)Primary highways. Two *highways* are depicted by the polygonal chains.

V. Multicast Schemes

Due to the overwhelming priority to access the spectrum, $\mathcal{N}_p(n)$ can operate as if no $\mathcal{N}_s(m)$ were present. Hence, we can design the multicast scheme for $\mathcal{N}_p(n)$ as the single ad hoc network. While, designing the strategy for SaN is a challenging issue, since we should maximize the throughput for SaN while ensuring the priority of PaN in terms of the order of throughput.

A. Schemes for Primary Network

We propose two types of multicast strategies for PaN called *percolation strategy* and *connectivity strategy*. The final *achievable throughput* will be derived by using cooperatively two types of strategies according to n_d and m_d.

1) Primary Percolation Strategy $\mathfrak{F}_p$: The strategy $\mathfrak{F}_p$ is based on two types of paths, *i.e.*, primary *percolation path* and primary *connectivity path*.

Primary Highway (PH): We concisely introduce the construction of primary *percolation paths*, *i.e.*, primary *highways* called in [8], and analyze the density of *highways* based on Lemma 5. We firstly partition the region $\mathcal{A}$ into subsquares with side length $l_p = \frac{c}{\sqrt{n}}$ as in Fig.1(a), where c is a constant, and call such subsquares *primary percolation cells* (PPCs). Then there are h_p^2 subsquares, where $h_p = \left\lceil \sqrt{n}/\sqrt{2}c \right\rceil$ (we can adjust the value of c such that $\sqrt{n}/\sqrt{2}c$ is an integer). Denote the lattice graph produced by inclined lines as $\mathbb{C}_p(h_p)$ (See Fig.1(a)). Let $N(c_i)$ denote the number of Poisson points inside cell c_i. Thus, for all i, the probability that a square c_i contains at least one Poisson point ($N(c_i) \geq 1$) is $p_p \equiv 1 - e^{-c^2}$. We say a square is open if it contains at least one point, and closed otherwise. Then any square is open with probability p_p, independently from each other. Then, we can map this model into a discrete *bond percolation model* on the square grid. Draw a horizontal edge across half of the squares, and a vertical edge across the others, as shown in Fig.1(b), by which we obtain the lattice graph $\mathbb{B}(h_p, p_p)$.

We say a given edge e in $\mathbb{B}(h_p, p_p)$ is open if the inclined subquare in $\mathbb{C}_p(h_p)$, crossed by e, is open. Based on an open crossing path connecting the left side of $\mathbb{B}(h_p, p_p)$ with its right side (or connecting the up side of $\mathbb{B}(h_p, p_p)$ with its bottom side), depicted in Fig.1(b). Choosing a node from each open cell in $\mathbb{C}_p(h_p)$ corresponding to each open edge of the open path and connect those nodes, we finally obtain a routing crossing path as in Fig.1(c). We call those nodes *stations* and call those routing crossing paths *highways* (or *percolation paths*). By Lemma 5, we have

Lemma 7: For any $\kappa > 0$ and $c^2 > \log 6 + 2/\kappa$, there exists a δ_p such that there are uniform *w.h.p.*, at least $\delta_p \log n$ horizontal (vertical) *highways* contained in all horizontal (vertical) slabs with sides of $1 \times \frac{\sqrt{2}c}{\sqrt{n}} \cdot (\kappa \log h_p - \epsilon(h_p))$.

Mapping from Highways to Slices: Partition each horizontal (or vertical) slab into $\delta_p \log n$ horizontal (or vertical) slices of width $w_p = \frac{\frac{\sqrt{2}c}{\sqrt{n}} \cdot (\kappa \log h_p - \epsilon(h_p))}{\delta_p \cdot \log n} = \Theta(\frac{1}{\sqrt{n}})$. By Lemma 7, we can assign at least one horizontal (or vertical) *highway* to each slices.

Algorithm 2 Primary Percolation Routing $\mathfrak{F}_p^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $EST(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: **for** each link $u_i \rightarrow u_j$ of $EST(\mathcal{U}_{\mathcal{S},k})$ **do**
2: u_i drains the packets into the specific horizontal *highway* along the specific *connectivity path*.
3: Packets are carried along the specific horizontal highway.
4: Packets are carried along the specific vertical highway.
5: Packets are delivered to u_j from the specific vertical highway along the specific *connectivity path*.
6: **end for**
7: Considering the resulted routing graph, we merge the same edges (hops), remove those circles which have no impact on the connectivity of the communications for $EST(\mathcal{U}_{\mathcal{S},k})$. Finally, we obtain $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

Primary Connectivity Path (PCP): Partition the region $\mathcal{A}$ into subsquares with side length $\bar{l}_p = \frac{\sqrt{\log n}}{\sqrt{n}}$ to obtain the lattice graph $\bar{\mathbb{C}}_p(\bar{h}_p)$. Then there are $\bar{h}_p^2$ subsquares, where $\bar{h}_p = \lceil \frac{n}{\log n} \rceil$, and we call them *connectivity cells*.

Lemma 8: All primary *connectivity cells* uniform *w.h.p.*, have at least one primary node.

Proof: Let N denote the number of nodes in a primary cell, then N follows a Poisson distribution with $\lambda = na_p = \log n$, then $\Pr(N = 0) = e^{-\lambda} = 1/n$. Thus, the probability that there is at least one cell having no node is upper bounded by $(n/\log n)\Pr(N = 0) = 1/\log n \rightarrow 0$, where union bounds and the fact there are $\Theta(n/\log n)$ cells are used. ■

Choose a node from each *connectivity cell* and connect them, we finally obtain the *connectivity path* (CPs). We call those nodes *connectivity stations*.

Primary Multicast Routing: Considering the multicast session $\mathcal{M}_{\mathcal{S},k}$, $1 \leq k \leq n_s$ and its *spanning set of nodes* $\mathcal{U}_{\mathcal{S},k}$. We firstly construct the Euclidean spanning tree (EST) spanning $\mathcal{U}_{\mathcal{S},k}$ by Algorithm 1. Based on $EST(\mathcal{U}_{\mathcal{S},k})$, we propose Algorithm 2 to construct the multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$. The routing scheme is with hierarchical structure consisting of the *highways phase* $\mathfrak{F}_p^{r_1}$ (including Steps in Line 3 and Line 4 of Algorithm 2) and *CPs phase* $\mathfrak{F}_p^{r_2}$ (including Steps in Line 2 and Line 5 of Algorithm 2).

Primary Transmissions Scheduling: We use two independent 9-TDMA schemes to schedule the highways and CPs, based on lattice graph $\mathbb{C}_p(h_p)$ and $\bar{\mathbb{C}}_p(\bar{h}_p)$ respectively. To be specific, we divide a scheduling period into two sub-periods with same size called *highway transmission scheduling* (HTS) $\mathfrak{F}_p^{t_1}$ and *CP transmission scheduling* (CPTS) $\mathfrak{F}_p^{t_2}$. The two scheduling phases correspond to the two phases of routing, *i.e.*, *highways phase* $\mathfrak{F}_p^{r_1}$ and *CPs phase* $\mathfrak{F}_p^{r_2}$. Furthermore, for each transmission in HTS phase, the transmitter transmits with power $P \cdot (l_p)^{\alpha}$, and in CPTS phase, the transmitter transmits with power $P \cdot (\bar{l}_p)^{\alpha}$.

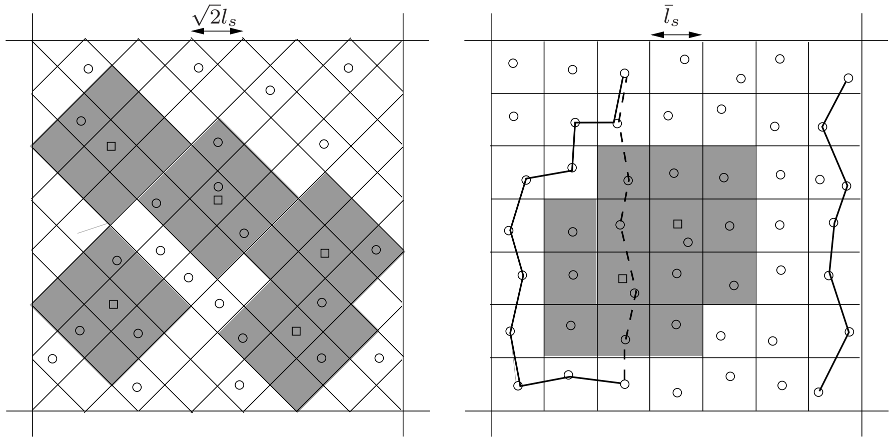
2) Primary Connectivity Strategy $\bar{\mathfrak{F}}_p$: Unlike the percolation strategy, the connectivity strategy only operates on the basis of CPs.

Primary Multicast Routing: We adopt Manhattan routing [5], [12], [18] based on CPs. (See detail in Algorithm 3)

Algorithm 3 Primary Connectivity Routing $\bar{\mathfrak{F}}_p^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $EST(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: **for** each link $u_i \rightarrow u_j$ of $EST(\mathcal{U}_{\mathcal{S},k})$ **do**
2: Denote the intersection point of the horizontal line through u_i and the vertical line through u_j as $p_{i,j}$.
3: Packets are carried along a specific horizontal CP from u_i to the *connectivity station* $u_{i,j}$ that locates in the *connectivity cell* containing point $p_{i,j}$.
4: Packets are carried along the specific vertical CP passing through $u_{i,j}$ to u_j.
5: **end for**
6: Use the similar way to Line 7 of Algorithm 2 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.



(a) Cognitive open SPCs (b) Secondary CPs

Fig. 2. (a) The shaded regions are the PPRs. The small square nodes at the center of PPRs represent the primary nodes, and the small circle nodes represent the secondary nodes. Those *secondary percolation cells* that contain at least one secondary nodes and are not shaded are *cognitive open*. (b) The shaded region represents the union of the CPRs.

Primary Transmissions Scheduling: For this case, since there are only links in CPs, we just only use CPTS.

B. Schemes for Secondary Network

The main challenge is how to design a protocol for $\mathcal{N}_s(m)$ such that $\mathcal{N}_s(m)$ achieve the optimal throughput, *i.e.*, the upper bound of the capacity. The key technique is to set *preservation regions* [19].

1) Secondary Percolation Strategy $\mathfrak{F}_s$: Like in $\mathcal{N}_p(n)$, there are two types of links in terms of hop length in $\mathcal{N}_s(m)$. The first type are the links along *secondary highways* with hop length $O(\frac{1}{\sqrt{m}})$, the second type are the links along *secondary connectivity paths* with hop length of order $O(\sqrt{\log m/m})$. To construct the routing consisting of two types of links above, as in $\mathcal{N}_p(n)$, we partition the region $\mathcal{A}$ into subsquares with side length $l_s = c/\sqrt{m}$ to obtain the lattice graph $\mathbb{C}_s(h_s)$ with $h_s = \lceil \frac{\sqrt{m}}{\sqrt{2}c} \rceil$, and we call the subsquares *secondary percolation cells* (SPCs). Similarly, divide $\mathcal{A}$ into subsquares with side length of $\bar{l}_s = \frac{\sqrt{\log m}}{\sqrt{m}}$ to obtain the *secondary connectivity cells* (SCCs) and lattice graph $\bar{\mathbb{C}}_s(\bar{h}_s)$ with $\bar{h}_s = \lceil \frac{\sqrt{m}}{\sqrt{\log m}} \rceil$.

Unlike in $\mathcal{N}_p(n)$, we must ensure that the secondary transmitters are not too close to the primary receivers operating simultaneously, otherwise, it may produce devastating interference. Hence, we set a *preservation region* for each primary nodes which the routing of communications in $\mathcal{N}_s(m)$ can not go through.

Preservation Region (PR): Based on lattice graphs $\mathbb{C}_s(h_s)$ and $\bar{\mathbb{C}}_s(\bar{h}_s)$, we define two types of *preservation regions*. The first is *percolation preservation region* (PPR) that consists of nine SPCs, with a primary node at the center cell. The second is *connectivity preservation region* (CPR) that consists of nine SCCs, with a primary node at the center cell.

Secondary Highway (SH): We construct the *secondary highways* based on lattice graph $\mathbb{C}_s(h_s)$. The difference from primary highways is that we must ensure the secondary highways not to pass through any PPRs. Thus, we should modify the definition of *open* for cells in $\mathbb{C}_s(h_s)$. We say a secondary percolation cell (SPC) is *cognitive open* if it is nonempty and does not belong to any PPRs. See illustration in Fig. 2(a). Then, we have

Lemma 9: When $n = o(m)$, a SPC in $\mathbb{C}_s(h_s)$ is *cognitive open* with probability p_s, where $p_s \to p_p$ as $n \to \infty$.

Proof: According to the definition of *cognitive open*, we have $p_s = (1 - e^{-c^2}) \cdot e^{-\frac{9c^2 n}{m}}$. Combining with the condition $\lim_{n\to\infty} \frac{n}{m} = 0$, we complete the proof. ■

By the similar procedure from $\mathbb{C}_p(h_p)$ to $\mathbb{B}(h_p, p_p)$, we can construct the lattice graph $\mathbb{B}(h_s, p_s)$ that serves as the basic frame of bond percolation model. By Lemma 5 and Lemma 7, we have

Lemma 10: When $n = o(m)$, for any $\kappa > 0$ and $c^2 > \log 6 + 2/\kappa$, there exists a constant δ_s such that there are uniform *w.h.p.*, at least $\delta_s \log m$ horizontal (vertical) *secondary highways* in each horizontal (vertical) slab with sides of $1 \times \frac{\sqrt{2}c}{\sqrt{m}}(\kappa \log h_s - \epsilon(h_s))$.

Mapping from Highways to Slices: Similar to PaN, we can partition each horizontal (or vertical) slab into $\delta_s \log m$ horizontal (or vertical) slices of width $w_s = \Theta(\frac{1}{\sqrt{m}})$. According to Lemma 7, we can assign at least one horizontal (or vertical) *highway* to each horizontal (or vertical) slice.

Secondary Connectivity Path (SCP): We can build the SCPs based on the lattice graph $\bar{\mathbb{C}}_s(\bar{h}_s)$. Similar to Lemma 8, we can obtain Lemma 11.

Lemma 11: All secondary *connectivity cells* (SCCs) uniform *w.h.p.*, have at least one secondary node.

Thus, we can choose a node from each SCC and connect them to obtain CPs-like paths. Notice that the difference between the primary *connectivity-paths* and the secondary ones is that we must ensure the SCPs not to pass through any CPR. Thus, we construct the SCPs by modifying the CPs-like paths in the following operations: When a CPs-like path collides with a CPR, the path detours the CPR along its boundary SCC, see Fig.2(b). We call all joint nodes on the SCPs *secondary connectivity stations* (SCSs).

Served Set: Unlike $\mathcal{N}_p(n)$, there are possibly some secondary cells (SPCs or SCCs) not to be served because they are covered by *preservation regions* or by the closed regions encompassed with *preservation regions* (PPRs or CPRs) clusters. We call those cells *non-served cells*, and define the set of all secondary nodes contained in the *non-served cells* as $\bar{\mathcal{V}}_s(m)$. Denote the set of all secondary sources for multicast sessions in $\mathcal{N}_s(m)$ as $\mathcal{S}$. (For succinctness, we denote both sets of sources of multicast sessions in $\mathcal{N}_p(n)$ and $\mathcal{N}_s(m)$ as $\mathcal{S}$ when having no confusion, but we should learn that they are really different sets.)

Based on the sets $\bar{\mathcal{V}}_s(m)$ and $\mathcal{S}$, we define the *served set* of multicast sessions. The definition of *served set* can be divided into two regions depending on m_d and m_s.

Definition 5 (Served Set): The *served set*, denoted as $\mathcal{S}'$, is a subset of $\mathcal{S}$, and

1) when $m_d = \omega(\log m_s)$,
   define $\mathcal{S}' := \mathcal{S} - \mathcal{S} \bigcap \bar{\mathcal{V}}_s(m)$;
2) when $m_d = O(\log m_s)$,
   define $\mathcal{S}' := \{v_{\mathcal{S},i} | \mathcal{U}_{\mathcal{S},i} \bigcap \bar{\mathcal{V}}_s(m) = \varnothing\}$.

In SaN $\mathcal{N}_s(m)$, for each multicast session $\mathcal{M}_{\mathcal{S},i}$ with source $v_{\mathcal{S},i} \in \mathcal{S}'$, define a set $\mathcal{U}'_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \bigcup \mathcal{D}'_{\mathcal{S},i}$, where $\mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i} - \mathcal{D}_{\mathcal{S},i} \bigcap \bar{\mathcal{V}}_s(m)$. In Section VI-A, we will prove that under Assumption B, $|\mathcal{S}'| \to |\mathcal{S}| = m_s$, and for all $v_{\mathcal{S},i} \in \mathcal{S}'$, uniform *w.h.p.*, $|\mathcal{D}'_{\mathcal{S},i}| \to |\mathcal{D}_{\mathcal{S},i}| = m_d$, as $n, m \to \infty$.

Secondary Multicast Routing: Based on every $\mathcal{U}'_{\mathcal{S},i}$ for $v_{\mathcal{S},i} \in \mathcal{S}'$, we can build $EST(\mathcal{U}'_{\mathcal{S},i})$ by Algorithm 1. The detailed routing scheme is presented in Algorithm 4.

Algorithm 4 Secondary Percolation Routing $\mathfrak{F}_s^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $EST(\mathcal{U}'_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

for each link $u_i \to u_j$ of $EST(\mathcal{U}'_{\mathcal{S},k})$ **do**
    u_i drains the packets into the specific horizontal SH along the specific SCP.
    Packets are carried along the horizontal SH.
    Packets are carried along along the vertical SH.
    Packets are delivered to u_j from the vertical SH along the specific SCP.
end for
Use the similar way to Line 7 of Algorithm 2 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

Secondary Transmissions Scheduling: To be synchronous with $\mathcal{N}_p(n)$, in $\mathcal{N}_s(m)$, the *secondary highways* in phase $\mathfrak{F}_s^{t_1}$ and SCPs in phase $\mathfrak{F}_s^{t_2}$ should be independently scheduled, where the length of each time slot in $\mathfrak{F}_s^{t_i}$ (for $i = 1, 2$) equals to that of $\mathfrak{F}_p^{t_i}$ but the scheduling periods of $\mathfrak{F}_s^{t_i}$ is 3 times of that of $\mathfrak{F}_p^{t_i}$. That is, we adopt two independent 27-TDMA schemes for *secondary highways* and SCPs, in which each secondary cell is scheduled for 3 continuous time slots in a scheduling period (27 time slots).

Furthermore, for each transmission in phase $\mathfrak{F}_s^{t_1}$, the transmitter transmits with power $P \cdot (l_s)^{\alpha}$, and in phase $\mathfrak{F}_s^{t_2}$, the transmitter transmits with power $P \cdot (\bar{l}_s)^{\alpha}$.

Algorithm 5 Secondary Connectivity Routing $\bar{\mathfrak{F}}_s^r$

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $EST(\mathcal{U}'_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

for each link $u_i \to u_j$ of $EST(\mathcal{U}'_{\mathcal{S},k})$ **do**
  Denote the intersection point of the horizontal line through u_i and the vertical line through u_j as $p_{i,j}$.
  Packets are carried along a specific horizontal SCP from u_i to the *connectivity station* $u_{i,j}$ that locates in the SCC containing point $p_{i,j}$.
  Packets are carried along the specific vertical SCP passing through $u_{i,j}$ to u_j.
end for
Use the similar way to Line 7 of Algorithm 2 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

2) Secondary Connectivity Strategy: We use Manhattan Routing [5], [12] based on SCPs system (Algorithm 5), and for this case, we only use the scheduling $\bar{\mathfrak{F}}_s^{t_2}$ since there are only links along SCPs to be scheduled.

C. Strategy Matchings

In terms of n_d and m_d, we will choose one better primary multicast strategy from $\mathfrak{F}_p$ and $\bar{\mathfrak{F}}_p$ for PaN, and determine one secondary multicast strategy from $\mathfrak{F}_s$ and $\bar{\mathfrak{F}}_s$ to match the selected primary multicast strategy.

STRATEGY ALTERNATIVES:

- When $n_d \in [1, \frac{n}{(\log n)^2}]$, we adopt $\mathfrak{F}_p$ for PaN.
  - When $m_d \in [1, \frac{m}{(\log m)^2}]$, we adopt $\mathfrak{F}_s$ for SaN.
  - When $m_d \in [\frac{m}{(\log m)^2}, m]$, we adopt $\bar{\mathfrak{F}}_s$ for SaN.
- When $n_d \in [\frac{n}{(\log n)^2}, n]$, we adopt $\bar{\mathfrak{F}}_p$ for PaN.
  - We always adopt $\mathfrak{F}_s$ for SaN.

VI. THROUGHPUT CAPACITY ANALYSIS

Since the definition of capacity (throughput) in [5], [12] can be regarded as the special cases of Definition 3, then we can derive the *achievable throughput* for PaN following the formal definition in [5], [12], as well as following Definition 3. For SaN, we consider the *achievable throughput* based on Definition 3, and we focus on the *served set* of sessions (Definition 5).

A. Analysis of Served Set

In this subsection, we aim to analyze the *served set* and mainly prove Lemma 12. We will discuss separately the issue for two cases when $m_d = \omega(\log m_s)$ and when $m_d = O(\log m_s)$.

Lemma 12: The cardinality of *served set* for $\mathcal{N}_s(m)$ defined in Definition 5, *i.e.*, $|\mathcal{S}'|$, goes to $|\mathcal{S}| = m_s$, and for all $v_{\mathcal{S},i} \in \mathcal{S}'$, uniform *w.h.p.*, $|\mathcal{D}'_{\mathcal{S},i}| \to |\mathcal{D}_{\mathcal{S},i}| = m_d$, as $n, m \to \infty$.

1) Total Area of Non-served Cells: Based on Lemma 4, we propose a lemma to show that the sizes of all clusters of preservation regions are bounded.

Lemma 13: When $n < \frac{\mathfrak{p}_c}{8} \cdot \frac{m}{\log m}$, any cluster of *preservation regions w.h.p.*, has at most a constant μ preservation regions, where $\mathfrak{p}_c$ is the *critical percolation threshold* of Poisson Boolean model in $\mathbb{R}^2$, m and n are the density of primary and secondary networks respectively.

Proof: First, we consider the Poisson Boolean model $\mathcal{B}(\lambda, r)$, where $r = 2\sqrt{2}\max\{l_s, \bar{l}_s\} = 2\sqrt{2} \cdot \frac{\sqrt{\log m}}{\sqrt{m}}$ and $\lambda = n$. Since the *associated graphs* $\mathcal{G}(\lambda_0, r_0) = \mathcal{G}(\lambda, r)$ when $\lambda_0 \cdot {r_0}^2 = \lambda \cdot r^2$. Hence, the Poisson Boolean model $\mathcal{B}(\lambda, r)$ is equivalent to $\mathcal{B}(\lambda_0, r_0)$ in terms of the connectivity, where $r_0 = 1$ and $\lambda_0 = 8n \cdot \frac{\log m}{m}$. Because $n < \frac{\mathfrak{p}_c}{8} \cdot \frac{m}{\log m}$, we have $\lambda \cdot r^2 < \mathfrak{p}_c$. By Lemma 4, the size of any cluster is at most a constant μ. Since a disk of radius $r/2$ contains a square preservation region, it is also true for all clusters of preservation regions. ■

Lemma 14: The sum area of the *non-served cells*, denoted as $S(m)$, is at most $9 \cdot \mu \cdot n \cdot \frac{\log m}{m}$, where the constant μ is the maximum size of the clusters of *preservation regions*.

Proof: For any cluster with size μ_i, it is true that there exists a square of side length $3\mu_i\bar{l}_s$ containing completely all μ_i *preservation regions* and the *non-served cells* encompassed by them. So, the sum area of the *non-served cells* produced by μ_i *preservation regions* $S(m,\mu_i) \leq 9\cdot\mu_i^2\cdot\frac{\log m}{m}$. Then, the sum area of the *non-served cells* $S(m) \leq S'_{max}$, where S_{max} is the optimum solution of the optimization problem:

$$\begin{cases} \max\ S = 9\cdot\frac{\log m}{m}\cdot\sum_{i=1}^{n}\mu_i^2 \\ \text{s.t.}\quad \sum_{i=1}^{n}\mu_i = n,\ 1\leq\mu_i\leq\mu,\ i=1,2,\cdots,n. \end{cases}$$

It is easy to derive that $S_{max} = \frac{n}{\mu}\cdot\mu^2\cdot 9\cdot\frac{\log m}{m} = 9\cdot\mu\cdot n\cdot\frac{\log m}{m}$, which completes the proof. ■

2) When $m_d = \omega(\log m_s)$: For this case, according to Definition 5, we have the *served set* $\mathcal{S}' = \mathcal{S} - \mathcal{S}\bigcap\bar{\mathcal{V}}_s(m)$. Then, we get $|\mathcal{S}'| = |\mathcal{S}| - |\mathcal{S}\bigcap\bar{\mathcal{V}}_s(m)|$. Notice that we need the condition that $n = o(m/\log m)$ made in Assumption B.

Lemma 15: With high probability, $|\mathcal{S}\bigcap\bar{\mathcal{V}}_s(m)| \leq \bar{\rho}_s(m)m_s$, where $\bar{\rho}_s(m)\to 0$, as $m\to\infty$.

Proof: Define a random variable $\bar{\xi}^s = |\mathcal{S}\bigcap\bar{\mathcal{V}}_s(m)|$. Then, it follows a poisson distribution with parameter $\bar{\lambda}^s \leq m_s\cdot S_{max} = 9\cdot\mu\cdot m_s\cdot n\cdot\frac{\log m}{m}$ by Lemma 14. According to Lemma 1, we get $\Pr(\bar{\xi}^s \geq 18\mu\cdot m_s\cdot n\cdot\log m/m) \leq (e/4)^{9\mu\cdot m_s n\cdot\frac{\log m}{m}} \to 0$. From $n = o(\frac{m}{\log m})$ (Assumption B), we can obtain $\bar{\rho}_s(m) = o(1)$. ■

Next, to derive the uniform upper bound of $|\mathcal{D}_{\mathcal{S},i}| - |\mathcal{D}'_{\mathcal{S},i}|$ for all $v_{\mathcal{S},i}\in\mathcal{S}'$, we firstly consider $|\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}|$.

Lemma 16: For all $v_{\mathcal{S},i}\in\mathcal{S}'$, uniform *w.h.p.*, $|\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}| \leq \bar{\rho}_d(m)\cdot m_d$, where $\bar{\rho}_d(m) = o(1)$.

Proof: For each $v_{\mathcal{S},i}\in\mathcal{S}'$, define a random variable $\bar{\xi}^d_{\mathcal{S},i} = |\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}|$. Then, according to Lemma 14, $\bar{\xi}^d_{\mathcal{S},i}$ follows a poisson distribution with mean of at most $9\cdot\mu\cdot m_d\cdot n\cdot\frac{\log m}{m}$. We consider separately two cases of $m_d\cdot n\cdot\frac{\log m}{m} = \Omega(\log m_s)$ and $m_d\cdot n\cdot\frac{\log m}{m} = O(\log m_s)$. Then, according to Lemma 1 (tails of Chernoff bounds) and union bounds, we can obtain

$$\bar{\rho}_d(m) = \begin{cases} O(n\cdot\frac{\log m}{m}) & \text{when } m_d\cdot n\cdot\frac{\log m}{m} = \Omega(\log m_s) \\ O(\frac{\log m_s}{m_d}) & \text{when } m_d\cdot n\cdot\frac{\log m}{m} = O(\log m_s) \end{cases}$$

Thus, we can get $\bar{\rho}_d(m) = o(1)$ when $m_d = \omega(\log m_s)$. ■

Combing Lemma 15 and Lemma 16, we can obtain Lemma 12 for the case when $m_d = \omega(\log m_s)$.

3) When $m_d = O(\log m_s)$: For this case, according to Definition 5, we have the *served set* $\mathcal{S}' = \{v_{\mathcal{S},i}|\,(v_{\mathcal{S},i}\in\mathcal{S})\wedge(\mathcal{U}_{\mathcal{S},i}\bigcap\bar{\mathcal{V}}_s(m) = \varnothing)\}$. Unlike in the case when $m_d = \omega(\log m_s)$, we need a new condition that $n = o(\frac{m}{m_d\cdot\log m})$ as in Assumption B. Firstly, we propose Lemma 17.

Lemma 17: For all $v_{\mathcal{S},i}\in\mathcal{S}'$, $\mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i}$.

Proof: According to the definition of $\mathcal{S}'$, for all $v_{\mathcal{S},i}\in\mathcal{S}'$, $\mathcal{U}_{\mathcal{S},i}\bigcap\bar{\mathcal{V}}_s(m) = \varnothing$. Since $\mathcal{D}_{\mathcal{S},i}\subseteq\mathcal{U}_{\mathcal{S},i}$, then $\mathcal{D}_{\mathcal{S},i}\bigcap\bar{\mathcal{V}}_s(m) = \varnothing$. Hence, $\mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i} - \mathcal{D}_{\mathcal{S},i}\bigcap\bar{\mathcal{V}}_s(m) = \mathcal{D}_{\mathcal{S},i}$. ■

Next, we consider the cardinality of $\mathcal{S}'$. Above all, we have that $|\mathcal{S}'| \geq |\mathcal{S}| - |\mathcal{S} - \mathcal{S}'|$.

Lemma 18: With high probability, $|\mathcal{S} - \mathcal{S}'| \leq \bar{\bar{\rho}}_s(m)\cdot m_s$, where $\bar{\bar{\rho}}_s(m) = o(1)$.

Proof: Define a random variable $\bar{\bar{\xi}}^s = |\mathcal{S} - \mathcal{S}'|$. Then by Lemma 14, $\bar{\bar{\xi}}^s$ follows a Poisson distribution with the mean of $\bar{\bar{\lambda}}^s \leq m_s\cdot(m_d+1)\cdot 9\cdot\mu\cdot n\cdot\log m/m$. By Chernoff bounds in Lemma 1, we get $\Pr(\bar{\bar{\lambda}}^s \geq 18m_s\cdot(m_d+1)\cdot\mu\cdot n\cdot\log m/m) \leq (\frac{e}{4})^{9m_s\cdot(m_d+1)\cdot\mu\cdot n\cdot\log m/m} \to 0$, as $m\to\infty$. By $n = o(\frac{m}{m_d\log m})$, we have $\bar{\bar{\rho}}_s(m) \leq 18m_s\cdot(m_d+1)\cdot\mu\cdot n\cdot\log m/m \to 0$. ■

Combining Lemma 17 and Lemma 18, we can obtain Lemma 12 for the case when $m_d = O(\log m_s)$.

4) Role of Served Set: Now, we discuss what role the *served set*, *i.e.*, $\mathcal{S}'$, will play. According to the routing schemes presented in Section V-B, only the sessions whose sources belong to the *served set* $\mathcal{S}'$ are considered, and for each considered session $\mathcal{M}_{\mathcal{S},i}$, only the destinations belong to $\mathcal{D}'_{\mathcal{S},i}$ are considered. Thus, by Lemma 12, we can state that the per-session throughput for SaN is achieved of λ bit/s if we can prove that, in $\mathcal{N}_s(m)$, for each multicast session $\mathcal{M}_{\mathcal{S},i}$ with source $v_{\mathcal{S},i}\in\mathcal{S}'$, uniform *w.h.p.*, data can be delivered to all destinations in $\mathcal{D}'_{\mathcal{S},i}$ at rate of λ bit/s. Here, obviously, $\mathcal{S}'$ can act as $\mathcal{S}'(1,1)$ in Definition 1.

B. Multicast Throughput Analysis

To facilitate the expression, for $\mathcal{N}_p(n)$, we define sets of a sequence of directed edges $\Pi_{\mathcal{S},k} = \{e_{ij}|\ e_{ij} = u_iu_j \in EST(\mathcal{U}_{\mathcal{S},k})\}$, where $k = 1,2,\cdots,n_s$; and for $\mathcal{N}_s(m)$, we define the sets $\Pi_{\mathcal{S},k} = \{e_{ij}|\ e_{ij} = u_iu_j \in EST(\mathcal{U}'_{\mathcal{S},k})\}$

for $v_{\mathcal{S},k} \in \mathcal{S}'$ (served set, defined in Section V-B). Recall that Event $E(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ in Definition 4, *i.e.*, multicast session $\mathcal{M}_{\mathcal{S},k}$ is routed through v_i^j based on the sub-routing scheme $\mathfrak{F}^{r_j}$ in Phase j. In the following analysis, to be succinct, we denote $\Pi_{\mathcal{S},k}$, $\mathcal{U}_{\mathcal{S},k}$ and $\mathcal{U}'_{\mathcal{S},k}$ as Π_k, $\mathcal{U}_k$ and $\mathcal{U}'_k$, denote $E(\mathfrak{F}^{r_j}, \mathcal{M}_{\mathcal{S},k}, v_i^j)$ as $E(j,k,i)$.

1) When $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [1, \frac{m}{(\log m)^2}]$: For this case, we implement $\mathfrak{F}_p$ for PaN and $\mathfrak{F}_s$ for SaN, and we analyze the throughput achieved by $\mathfrak{F}_p^r$ with $\mathfrak{F}_s^r$ according to Lemma 6. First, we consider phase $\mathfrak{F}_p^{r_1}$ (or $\mathfrak{F}_s^{t_1}$).

Lemma 19: During phase $\mathfrak{F}_p^{t_1}$ (or phase $\mathfrak{F}_s^{t_1}$), the total rate along *highways* (including primary *highways* in $\mathcal{N}_p(n)$ and secondary highways in $\mathcal{N}_s(m)$) is achieved of order $\Omega(1)$.

Please see the proof in Appendix.

For the multicast throughput during highways phases, according to Lemma 6, we have the following result.

Lemma 20: During phase $\mathfrak{F}_p^{r_1}$ (or phase $\mathfrak{F}_s^{r_1}$), P-MPMT (or S-MPMT) is achieved of order $\Omega(\frac{1}{\sqrt{nn_d}})$ (or $\Omega(\frac{1}{\sqrt{mm_d}})$).

Next, we analyze the throughput in phase $\mathfrak{F}_p^{r_2}$ (or $\mathfrak{F}_s^{r_2}$).

Lemma 21: During phase $\mathfrak{F}_p^{r_2}$ (or $\mathfrak{F}_s^{r_2}$), P-MPMT (or S-MPMT) is achieved of order $\Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$ (or $\Omega(\frac{1}{m_d} \cdot (\log m)^{-\frac{3}{2}})$).

Combining Lemma 20 and Lemma 21, we get Theorem 1.

2) When $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [\frac{m}{(\log m)^2}, m]$: For this case, we implement $\mathfrak{F}_p$ for PaN and $\bar{\mathfrak{F}}_s$ for SaN. In phase $\mathfrak{F}_p^{r_1}$, we can set transmissions in SaN be idle, which can not have impact on throughput in order sense. Then, it is obviously true that the throughput for PaN during phase $\mathfrak{F}_p^{r_1}$ for this case is no less than that for the previous case. Furthermore, during phase $\mathfrak{F}_p^{r_2}$, we implement strategy $\bar{\mathfrak{F}}_s$ for SaN. Using a similar method in Lemma 21, we can get that the interference produced by $\bar{\mathfrak{F}}_s$ to transmissions of PaN is no more than our estimation (in Lemma 21) of that produced by $\mathfrak{F}_s^{t_1}$. Summing the analysis above, we can easily obtain

Lemma 22: During phase $\mathfrak{F}_p^{r_1}$ (or $\mathfrak{F}_p^{r_2}$), P-MPMT can be achieved of order $\Omega(\frac{1}{\sqrt{nn_d}})$ (or $\Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$).

By Lemma 22, it is obviously true that P-MPMT is achieved of order $T_1(n, n_d)$ when $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [\frac{m}{(\log m)^2}, m]$. Next, we consider the throughput for SaN.

Lemma 23: S-MPMT can be achieved of order $T_2(m, m_d)$, where the function $\mathrm{T}_2(x,y)$ is defined in Section III.

Combining Lemma 22 and Lemma 23, we get Theorem 2.

3) When $n_d \in [\frac{n}{(\log n)^2}, n]$: For this case, we adopt $\bar{\mathfrak{F}}_p$ for PaN and adopt $\bar{\mathfrak{F}}_s$ for SaN. In a similar way to Lemma 23, we can obtain Theorem 3 due to the non-hierarchical structure of strategies $\bar{\mathfrak{F}}_p$ and $\bar{\mathfrak{F}}_s$.

VII. Literature Review

Capacity for Single Ad hoc Networks: In [20], Gupta and Kumar showed for *unicast* sessions, each source-destination (S-D) pair can achieve a rate of order $1/\sqrt{n \log n}$ in *random dense networks*. Keshavarz-Haddad, et al. [21] showed the broadcast per-session capacity is only of order $\Theta(1/n)$. Li *et al.* [5] showed that, for *random networks*, the per-session multicast capacity is $\Theta(1/\sqrt{n_d n \log n})$ when $n_d = O(n/\log n)$, and is $\Theta(1/n)$ when $n_d = \Omega(n/\log n)$ under the assumption $n_s = n$. Shakkottai *et al.* designed a novel routing scheme, called *comb scheme*. Notice that all above results are derived under the *protocol model* or *physical model* [20]. For the *Gaussian Channel* model that captures better the property of physical layer in wireless networks, some representative works recently have been carried out . Franceschetti *et al.* [8] showed that the unicast throughput of $\Omega(1/\sqrt{n})$ is achievable in random networks using *percolation model*. Zheng [11] pointed out that using multihop relay, the broadcast per-session capacity is $\Theta(\frac{1}{n} \cdot (\log n)^{-\frac{\alpha}{2}})$ in *random extended networks*. Li et al. [9] proposed that, for *random extended networks*, when $n_d = O(\frac{n}{(\log n)^{2\alpha+6}})$, the achievable per-session multicast throughput is *w.h.p.*of order $\Omega(1/\sqrt{nn_d})$ using *bond percolation model* [8], [15]. In [22], Wang *et al.* improve the threshold value of n_d mentioned in [9] to $n_d = O(\frac{n}{(\log n)^{\alpha+1}})$. For *dense networks*, by a technique called *arena*, Keshavarz-Haddad *et al.* ([6], [14]) propose an upper bound on multicast capacity, and sketched a routing with the estimation of the achievable throughput.

Capacity for Cognitive Networks: The research on capacity scaling laws for cognitive networks is a relatively new topic. In [3], the primary source-destination and cognitive S-D pairs are modeled as an interference channel with asymmetric side information. In [23] the communication opportunities are modeled as a two-switch channel. Note that both work [3], [23] has only considered the *single-user* case in which a single primary and a single cognitive S-D pairs share the spectrum. Recently, a *single-hop* cognitive network was considered in [24], where multiple secondary S-D pairs transmit in the presence of a single primary S-D pair. They showed that a linear scaling law of the *single-hop* secondary network is obtained when its operation is constrained to guarantee a particular outage constraint for the primary S-D pair. For multi-hop and multiple users case, the most related work to this paper is done by Jeon et al. [7]. They focused on unicast sessions in a scenario where the primary networks and secondary networks co-exist. They only adopted the routing similar to the *connectivity strategies*, which results the derived throughput is not optimal under the *Gaussian Channel* model.

VIII. Conclusion and Future Work

We study the multicast capacity of the cognitive network consisting of the primary ad hoc network and secondary ad hoc network. We show that under some conditions, we can design the corresponding strategies to let both networks achieve the asymptotic capacity as they are stand-alone. As a future work, we would like to extend our results to the case when the *primary network* is an infrastructure-supported network (cellular network) or hybrid network [25], [26]. Another interesting and significant issue is to extend our results derived under *dense networks* into ones derived under *extended networks*. Furthermore, as for single ad hoc network, the most challenging question is to close the remaining gap between the lower bounds and upper bounds on multicast capacity by presenting possibly new tight upper bounds or designing algorithms to achieve the asymptotic multicast capacity.

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Appendix

Proof of Lemma 6: Given the total achievable rate sustained by every node in $\mathcal{V}(\mathfrak{F}^{r_j})$, denoted as R_j, we need only consider the uniform upper bound on the relay burden of all nodes in $\mathcal{V}(\mathfrak{F}^{r_j})$ during Phase j, denoted as L_j.
First, define the relay burden of v_i^j, *i.e.*, the number of multicast session routed through v_i^j during Phase j, as X_i^j; and define the uniform upper bound of X_i^j for all $v_i^j \in \mathcal{V}(\mathfrak{F}^{r_j})$ as X_j. By Eq. (4), we have that an upper bound of X_i^j, denoted as Y_i^j, follows a Poisson distribution with $\lambda_j = n_s \cdot Q_j$. Hence, by union bounds, we have that $\Pr(X_j \geq x) \leq |\mathcal{V}(\mathfrak{F}^{r_j})| \cdot \Pr(X_i^j \geq x) \leq n \Pr(Y_i^j \geq x)$.
Second, We upperbound X_j. By $Q_j = \Omega(\frac{\log n}{n})$, we have $\lambda_j = \Omega(\log n)$. Based on a constant Δ_1 satisfying $\lambda_j > \Delta_1 \cdot \log n$, there exists a large enough constant Δ_2 such that

$$\Delta_2 \cdot (\log \Delta_2 - 1) + 1 > 1/\Delta_1 \tag{5}$$

According to Lemma 1, we have

$$\Pr(X_j \geq \Delta_2 \cdot \lambda_j) \leq n \cdot \Pr(Y_i^j \geq \Delta_2 \cdot \lambda_j) \leq \ n \cdot \left(\frac{e^{\Delta_2}}{e \cdot \Delta_2^{\Delta_2}}\right)^{\Delta_1 \log n} = n^{1-\Delta_1 \cdot (\Delta_2 \cdot (\log \Delta_2 - 1)+1)} \tag{6}$$

Combing Eq. (5) and Eq. (6), we have that $\Pr(X_j \geq \Delta_2 \lambda_j) \to 0,$ as $n \to 0$. Hence, we can obtain that $Load_j = O(n_s \cdot Q_j)$. Combining with $\Lambda_j = R_j/Load_j$, we complete the proof. ∎

Proof of Lemma 19: Part 1: In this part, we analyze the rate along primary highways. Consider any transmitter v_i^p and its receiver v_j^p, where $v_i^p, v_j^p \in \mathcal{V}(\mathfrak{F}_p^{r_1})$ (See Definition 4). Thus, v_i^p and v_j^p locate on two adjacent PPCs.

First, we bound $\mathrm{I}_{pp}(v_i^p, v_j^p)$. We observe that the transmitters in the eight closest cells are located at Euclidean distance at least l_p. *i.e.*, $\frac{c}{\sqrt{n}}$ from the receiver v_j^p. The 16 next closest cells are at Euclidean distance at least $4l_p$. By extending the sum of the interferences to the whole region, it can be bounded above as $\mathrm{I}_{pp}(v_i^p, v_j^p) \leq \sum_{i=1}^{\lceil 1/l_p^2 \rceil} 8iP(l_p)^\alpha \ell((3i-2)l_p) \leq P \sum_{i=1}^{n} \frac{8i}{(3i-2)^\alpha}$ Since $\alpha > 2$, by the *Cauchy Test*, $\lim_{n\to\infty} \mathrm{I}_{pp}(v_i^p, v_j^p) \leq 8P \cdot \Delta_3(\alpha)$, where $\Delta_3(\alpha)$ is a constant depending on α.

Second, we upperbound $\mathrm{I}_{sp}(v_i^p, v_j^p)$. The *percolation preservation region* (PPR) centered on v_j^p consists of 9 SPCs, hence, for any slot τ, there must exist one SPC out of the 9 SPCs that would be scheduled in τ if it was not in the *preservation region*. We denote the cell as c_τ. Considering those cells containing the nodes in $\mathcal{V}_s(v_i^p, \tau)$, it can be seen that the secondary users in the eight closest cells to c_τ are far away from v_j^p with distance at least l_s, *i.e.*, $\frac{c}{\sqrt{m}}$. Thus, the sum of the interferences can be upperbounded as $\mathrm{I}_{sp}(v_i^p, v_j^p) \leq 8P \sum_{i=1}^{\infty} i(3i-2)^{-\alpha}$. Then, we can obtain that $\lim_{n\to\infty} \mathrm{I}_{sp}(v_i^p, v_j^p) \leq 8P \cdot \Delta_4(\alpha)$, where $\Delta_4(\alpha)$ is a constant depending on α..

Third, we lower bound the signal received from the transmitter $\mathrm{S}(v_i^p, v_j^p)$. Since any communication pairs locate in the adjacent cell, *i.e.*, $\|v_i^p v_j^p\| \leq \sqrt{5} \cdot l_p$, the strength of the signal is $\mathrm{S}(v_i^p, v_j^p) \geq P(l_p)^\alpha (\sqrt{5} \cdot l_p)^{-\alpha} = 5^{-\frac{\alpha}{2}} P$.

Finally, we consider the limit of SINR, we obtain $\mathrm{R}_p(v_i^p, v_j^p) \geq \log(1 + \frac{5^{\alpha/2}}{N_0/P + 8\Delta_3(\alpha) + 8\Delta_4(\alpha)}) \geq R_1$, where $R_1 > 0$ is a constant. Since each transmitter in $\mathcal{V}(\mathfrak{F}_p^{r_1})$ is scheduled at least once out of 9 time slots, we have the total rate along the primary *highways* is achieved of at least $\frac{1}{9} R_1$, which proves the result for PaN.

Part 2: In this part, we analyze the achievable rate along secondary highways. For any link $v_i^s v_j^s$ in $\mathcal{N}_s(m)$, if v_j^s is out of the PPRs it can be served. However, there is possibly a time slot τ_0 in which the distance from a primary node $v_0^p \in \mathcal{V}_p(v_i^s, \tau_0)$ to v_j^s is so close that a fatal interference is imposed on v_j^s. The secondary transmissions scheduling scheme $\mathfrak{F}_s^{t_1}$ can prevent this scenario. Since for $\mathcal{N}_s(m)$ the same packet is transmitted along 3 time slots, we can guarantee that there exists a slot τ out of 3 time-slots in which the minimum distance to v_j^s from all $v^p \in \mathcal{V}_p(v_i^s, \tau)$ is at least $\frac{l_p}{2}$. Then, $\mathrm{I}_{ps}(v_i^s, v_j^s) < \sum_{i=1}^{\infty} 8iP(l_p)^\alpha \ell((3i-2)l_p) + P(l_p)^\alpha \ell(\frac{l_p}{2})$. Hence, $\mathrm{I}_{ps}(v_i^p, v_j^p) < 8P \cdot \Delta_3(\alpha) + 2^\alpha \cdot P$. In a similar way to Part 1, we obtain that $\mathrm{I}_{ss}(v_i^s, v_j^s) < 8P \cdot \Delta_3(\alpha)$. Combining with $S(v_i^s, v_j^s) \geq P \cdot (l_s)^\alpha \cdot (\sqrt{5} l_s)^{-\alpha}) = 5^{-\frac{\alpha}{2}} \cdot P$, we can obtain that $R_s(v_i^s, v_j^s) \geq \log\left(1 + \frac{5^{-\frac{\alpha}{2}} \cdot P}{N_0 + 16P\Delta_3(\alpha) + 2^\alpha \cdot P}\right) \geq R_2$, where $R_2 > 0$ is a constant. Since each transmitter in $\mathcal{V}(\mathfrak{F}_s^{r_1})$ is successfully scheduled at least once out of 27 time slots, we have the total rate along the secondary *highways* is achieved of at least $\frac{1}{27} R_2$, which completes the proof. ∎

Proof of Lemma 20: We derive the result for PaN and extend the result to that for SaN. According to Lemma 19, the rate of all *stations* on the primary highways can be achieved of a constant order during phase $\mathfrak{F}_p^{t_1}$, *i.e.*, $R_1^p = \Omega(1)$. By Lemma 6, we only need prove the area of *sufficient regions* of all primary *stations* are uniform *w.h.p.*, at most $Q_1^p = O(\frac{\sqrt{n_d}}{\sqrt{n}})$.

Given a *station* v_t^p passed by a primary *highway*. First, we analyze Event $E(1,k,t)$, *i.e.*, $E(\mathfrak{F}_p^{r_1}, \mathcal{M}_k, v_t^p)$. For $e_{ij} \in \Pi_k$, define Event $E_{ij}(1,k,t)$ as: During phase $\mathfrak{F}_p^{r_1}$, the routing from u_i to u_j passes by v_t^p. Obviously, we have that $\Pr(E(1,k,t)) = \Pr(\bigcup_{e_{ij}\in\Pi_k} E_{ij}(1,k,t))$. According to union bounds, we obtain

$$\Pr(E(1,k,t)) \leq \min\left\{\sum\nolimits_{e_{ij}\in\Pi_k} \Pr(E_{ij}(1,k,t)), 1\right\}.$$

Second, we upperbound $\Pr(E_{ij}(1,k,t))$. Considering the routing of $u_i \to u_j$ in phase $\mathfrak{F}_p^{r_1}$, define the horizontal (or vertical) Euclidean distance at which data are transmitted as *horizontal (or vertical) span distance* $L_{ij}^{p,h}$ (or $L_{ij}^{p,v}$). Then,

$$L_{ij}^{p,h} \leq \|u_i u_{i,j}\| + \varpi(n), \quad L_{ij}^{p,v} \leq \|u_{i,j} u_j\| + \varpi(n)$$

where the point $u_{i,j}$ is determined in Algorithm 2, $h_p = \left\lceil \frac{\sqrt{n}}{\sqrt{2}c} \right\rceil$ and $\varpi(n) = \frac{\sqrt{2}c}{\sqrt{n}} \cdot (\kappa \log h_p - \epsilon(h_p) + 1) + \frac{\sqrt{\log n}}{\sqrt{n}}$. Consider a square region $\mathcal{Q}_{ij}(1,k,t)$ with sides of $w_p \times 2(L_{ij}^{p,h} + L_{ij}^{p,v})$ (Recall w_p is the width of the slice), we have,

$$\Pr(E_{ij}(1,k,t)) \leq \Pr(u_i \text{ locates in } \mathcal{Q}_{ij}(1,k,t))$$

Finally, we upperbound $\|\mathcal{Q}(1,k,t)\|$ as Q_1^p. Straightforwardly, $\|\mathcal{Q}(1,k,t)\| \leq \sum_{e_{ij}\in\Pi_k} 2w_p \cdot (L_{ij}^{p,h} + L_{ij}^{p,v})$. By Lemma 2, we have that $\|EST(\mathcal{U}_{\mathcal{S},k})\| = \sum_{e_{ij}\in\Pi_k} \|u_i u_j\| < 2\sqrt{2} \cdot \sqrt{n_d}$. Since $\|u_i u_{i,j}\| + \|u_{i,j} u_j\| \leq \sqrt{2}\|u_i u_j\|$, we have

$$\|\mathcal{Q}(1,k,t)\| = O(\sqrt{n_d}/\sqrt{n} + n_d \log n / n) \tag{7}$$

Since $n_d = O(n/(\log n)^2)$, *i.e.*, $\sqrt{n_d}/\sqrt{n} = \Omega(n_d \log n/n)$, $Q_1^p = O(\sqrt{n_d}/\sqrt{n})$. Then, we proves the result for PaN. By a similar procedure, we can prove the result for SaN. ■

Proof of Lemma 21: Using a similar procedure of Lemma 19, we can obtain during phase $\mathfrak{F}_p^{t_2}$ (or phase $\mathfrak{F}_s^{t_2}$), the total rate along PCPs (or SCPs) can be achieved of order $\Omega(1)$. By Lemma 6, we need only prove that the area of *sufficient regions* of $v_t^p \in \mathcal{V}(\mathfrak{F}_p^{r_2})$ (or $v_t^s \in \mathcal{V}(\mathfrak{F}_s^{r_2})$) are uniform *w.h.p.*, at most of $Q_2^p = O(\frac{n_d}{n} \cdot (\log n)^{3/2})$(or $Q_2^s = O(\frac{m_d}{m} \cdot (\log m)^{3/2})$).

Part 1: (for PaN) Given a primary node $v_t^p \in \mathcal{V}(\mathfrak{F}_p^{r_2})$, we firstly consider Event $E(2,k,t)$, *i.e.*, $E(\mathfrak{F}_p^{r_2}, \mathcal{M}_k, v_t^p)$. For $e_{ij} \in \Pi_k$, define Event $E_{ij}(2,k,t)$ as: During phase $\mathfrak{F}_p^{r_2}$, the routing from u_i to u_j passes through v_t^p. Hence, we have

$$\Pr(E(2,k,t)) \leq \min\left\{n_d \cdot \max\nolimits_{e_{ij}\in\Pi_k}\{\Pr(E_{ij}(2,k,t))\}, 1\right\}.$$

Next, we consider the bounds of $\Pr(E_{ij}(2,k,t))$. Considering phase $\mathfrak{F}_p^{r_2}$ of routing for $u_i \to u_j$, denote the maximum horizontal (or vertical) distance along which data are transmitted horizontally (or vertically) as $\bar{L}_{ij}^{p,h}$ (or $\bar{L}_{ij}^{p,v}$). Then,

$$\bar{L}_{ij}^{p,h} \leq \frac{\sqrt{2}c}{\sqrt{n}} \cdot \kappa \cdot \log h_p + \bar{l}_p, \qquad \bar{L}_{ij}^{p,v} \leq \frac{\sqrt{2}c}{\sqrt{n}} \cdot \kappa \cdot \log h_p + \bar{l}_p$$

where $h_p = \left\lceil \frac{\sqrt{n}}{\sqrt{2}c} \right\rceil$ and $\bar{l}_p = \frac{\sqrt{\log n}}{\sqrt{n}}$. Consider a square region $\mathcal{Q}_{ij}(2,k,t)$ with sides of $\bar{l}_p \times (\bar{L}_{ij}^{p,h} + \bar{L}_{ij}^{p,v})$, then we have $\|\mathcal{Q}_{ij}(2,k,t)\| = O(\frac{(\log n)^{3/2}}{n})$. Furthermore, we have

$$\Pr(E_{ij}(2,k,t)) \leq \Pr(u_i \text{ locates in } \mathcal{Q}_{ij}(2,k,t)).$$

Thus, $\|\mathcal{Q}(2,k,t)\| \leq n_d \max_{e_{ij}\in\Pi_k}\{\|\mathcal{Q}_{ij}(2,k,t)\|\} = O(\frac{n_d \cdot (\log n)^{\frac{3}{2}}}{n})$. Then, we can choose $Q_2^p = O(\frac{n_d \cdot (\log n)^{3/2}}{n})$, which proves the result for PaN according to Lemma 6.

Part 2: (for SaN) In the following procedure, a significant difference from Part 1 is that the routing paths cannot possibly extend directly along horizontal or vertical lines due to the block of *preservation regions*. The routing would detour around the *preservation regions*, which should increase the transmitting distance of the data and the area of *sufficient region* for some nodes. We mainly prove that the area of *sufficient region* for all nodes $v_t^s \in \mathcal{V}(\mathfrak{F}_s^{r_2})$ does not increase in order sense. For any secondary node $v_t^s \in \mathcal{V}(\mathfrak{F}_s^{r_2})$, we firstly consider Event $E_s(2,k,t)$, *i.e.*, $E(\mathfrak{F}_s^{r_2}, \mathcal{M}_k, v_t^s)$. For any edge $e_{ij} = u_i u_j \in \Pi_k$, define Event $E_{ij}^s(2,k,t)$ as: During phase $\mathfrak{F}_s^{r_2}$, the routing from u_i to u_j passes through v_t^s. Next, we aim to construct a region $\mathcal{Q}_{ij}^s(2,k,t)$ such that

$$\Pr(E_{ij}^s(2,k,t)) \leq \Pr(u_i \text{ locates in } \mathcal{Q}_{ij}^s(2,k,t)) \tag{8}$$

Centered at the SCC containing v_t^s, the region $\mathcal{Q}_{ij}^s(2,k,t)$ has sides of $3 \cdot \mu \cdot \bar{l}_s \times \frac{\sqrt{2}c}{\sqrt{m}} \cdot (\kappa \log h_s - \epsilon(h_s))$. Easily, we can get $\|\mathcal{Q}_{ij}^s(2,k,t)\| = O(\frac{(\log m)^{3/2}}{m})$. By Lemma 13, Eq. (8) is true. Hence, we obtain $\|E_s(2,k,t)\| = O(m_d \cdot \frac{(\log m)^{3/2}}{m})$, and complete the proof. ■

Proof of Lemma 23: Similar to Lemma 21, we obtain under the strategy $\bar{\mathfrak{F}}_s$, the total rate along SCPs can be achieved of order $\Omega(1)$. Then, we only need to prove that the area of *sufficient region* of all nodes $\bar{v}_t^s \in \mathcal{V}(\bar{\mathfrak{F}}_s^r)$ is bounded above by

$$\bar{Q}^s = \begin{cases} O(\sqrt{m_d \cdot \frac{\log m}{m}}) & \text{when } m_d \sim [1, \frac{m}{\log m}] \\ O(1) & \text{when } m_d \sim [\frac{m}{\log m}, m] \end{cases} \tag{9}$$

Define Event $\bar{E}^s(1,k,t)$ as: During $\bar{\mathfrak{F}}_s^r$, $\mathcal{M}_{\mathcal{S},k}$ passes through $\bar{v}_t^s$, and for any $e_{ij} = u_i u_j \in \Pi_k$, define Event $\bar{E}_{ij}^s(1,k,t)$ as: Under $\bar{\mathfrak{F}}_s^r$, the routing from u_i to u_j passes through $\bar{v}_t^s$. Naturally, we should construct a region $\bar{\mathcal{Q}}_{ij}^s(1,k,t)$ such that

$$\Pr(\bar{E}_{ij}^s(1,k,t)) \leq \Pr(u_i \text{ locates in } \bar{\mathcal{Q}}_{ij}^s(1,k,t)) \tag{10}$$

Similar to Part 2 of Lemma 21, we must make the routing path detour the blocking *connectivity preservation regions*. Thus, to ensure Eq. (10) to be true, we build the region $\bar{\mathcal{Q}}_{ij}^s(1,k,t)$ as a square with sides of $3 \cdot \mu \cdot \bar{l}_s \times 2 \cdot (\|u_i u_{i,j}\| + \|u_{i,j} u_j\| + 2 \cdot \bar{l}_s)$ centering at the SCC containing $\bar{v}_t^s$. Hence, we have $\|\bar{\mathcal{Q}}^s(1,k,t)\| \leq \min\left\{\sum_{e_{ij} \in \Pi_k} \|\bar{\mathcal{Q}}_{ij}^s(1,k,t)\|, 1\right\}$. By Lemma 2, we can obtain $\|\bar{\mathcal{Q}}^s(1,k,t)\| = O(\sqrt{m_d \cdot \frac{\log m}{m}} + m_d \cdot \frac{\log m}{m})$. Choose $\bar{Q}^s$ as Eq. (9), we complete the proof. ■