OURS: Optimal Unicast Routing Systems in Non-Cooperative Wireless Networks

Weizhao Wang* Stephan Eidenbenz† Yu Wang‡ Xiang-Yang Li*

* Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616, USA
† Discrete Simulation Science, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
‡ Department of Computer Science, University of North Carolina at Charlotte, Charlotte, NC 28223, USA

ABSTRACT

We propose novel solutions for unicast routing in wireless networks consisted of selfish terminals: in order to alleviate the inevitable over-payment problem (and thus economic inefficiency) of the VCG (Vickrey-Clark-Groves) mechanism, we design a mechanism that results in Nash equilibria rather than the traditional strategyproofness (using weakly dominant strategy). In addition, we systematically study the *unicast routing system* in which both the relay terminals and the service requestor (either the source or the destination nodes or both) could be selfish. To the best of our knowledge, this is the *first* paper that presents *social efficient* unicast routing systems with proved performance guarantee. Thus, we call the proposed systems: *O*ptimal *U*nicast *R*outing *S*ystems (OURS).

Our main contributions of OURS are as follows. (1) For the principal model where the service requestor is not selfish, we propose a mechanism that provably creates incentives for intermediate terminals to cooperate in forwarding packets for others. Our mechanism substantially reduces the overpayment by using Nash equilibrium solutions as opposed to strategyproof solutions. We then study a more realistic case where the service requestor can act selfishly. (2) We first show that if we insist on the requirement of strategyproofness for the relay terminals, then no system can guarantee that the central authority can retrieve at least $\frac{1}{n}$ of the total payment. (3) We then present a strategyproof unicast system that collects $\frac{1}{2n}$ of the total payment, which is thus asymptotically optimum. (4) By only requiring Nash Equilibrium solutions, we propose a system that creates incentives for the service requestor and intermediate terminals to correctly follow the prescribed protocol. More importantly, the central authority can retrieve at least half the total payment. We verify the economic efficiency of our systems through simulations that are based on very realistic terminal distributions.

*This work was partially done when Weizhao Wang interned at the Los Alamos National Lab, and he is now with Google Inc. This work of Xiang-Yang Li was partially supported by NSF CCR-0311174.

†Los Alamos Technical Report No. LA-UR-06:2166

‡This work of Yu Wang was supported, in part, by funds provided by Oak Ridge Associated Universities.

MobiCom'06, September 23–26, 2006, Los Angeles, California, USA.

Categories and Subject Descriptors

C.2.2 [**Network Protocols**]: Routing Protocols; G.2.2 [**Graph Theory**]: Network problems, Graph algorithms.

General Terms

Algorithms, Design, Economics, Theory.

Keywords

Wireless networks, game theory, mechanism design, Nash Equilibrium, dominant strategy, frugality ratio, non-cooperative.

1. INTRODUCTION

Wireless networks continue to gain importance with wide-spread demand for high-bandwidth applications such as multimedia streaming in ad hoc environments. The past years have seen a considerable amount of research in wireless networks on various aspects including routing, quality of service, security, power management, and traffic and mobility modelling. The vast majority of these protocols rely on the assumption that each wireless terminal will follow the prescribed protocol without any deviation. However, this assumption of cooperation is not true in many application scenarios [1, 4, 20, 22, 25]. In many scenarios, the wireless terminals are owned by individual profit-maximizing entities such as human users who will cooperate only if it suits their needs and interests. In practice, wireless terminals are often powered by batteries, thus it is not in the best interest of a wireless terminal to always forward data packets for other terminals as doing so depletes its battery without providing utility to its owner. Thus, the most rational strategy for an individual user is to always turn off its device except when the user wants to participate in a communication as source or destination; such behavior in turn can be detrimental to network performance, and in the case of ad hoc networks it inevitably leads to disconnected networks. In order to overcome this strategy, stimulation mechanisms are required to encourage users to provide service to other terminals.

Two different approaches for dealing with selfish behavior have been proposed by the research community: reputation-based systems [3–5,12,20] keep records of the cooperative behavior and punish non-cooperating terminals; incentive-based systems [1, 22, 25] actively pay terminals for collaboration. In this paper, we pursue an incentive-based approach that allows us to apply and extend concepts from game theory, which results in provable properties. The key issue that any incentive-based system needs to solve is how much terminals should be paid for forwarding data packets such that the system reaches kind of steadiness. One of the usual approaches so far is to induce terminals to tell the truth with regards to their cost. A system or mechanism that achieves this is called *strategyproof*. The best-known mechanism that can induce strategyproofness is the VCG(Vickrey-Clark-Groves) mechanism. Anderegg *et al.* [1] have proposed a routing protocol for unicast routing in a wireless ad hoc network based on the VCG mechanism. Wang *et al.* [22] have generalized the result and proposed some non-VCG mechanism for multicast routing in wireless ad hoc networks. In [25], Zhong *et al.* presents an alternate view of the selfish routing problem that very elegantly distinguishes between a routing and an actual forwarding phase.

Notice that none of these systems showed how to obtain the money to pay for the relay terminals. One natural explanation is that these money is collected from the *service requestor* who initiate or benefit from the routing, *e.g.*, source node or destination node or both. However, it is not likely that the service requestor is always willing to pay the money to relay terminals when it is arbitrarily large. Thus, we assume that the service requestor has a *budget constraint* w to indicate how much it is wiling to pay for the service. Therefore, the whole system we study is composed of not only the relay terminals but also the service requestor, who may also be selfish and falsely report its budget constraint. A good illustration of such system is a commercial WIFI scenario shown in Figure 1. In the system, one node (big square) acts as the destination, which could either be a *service access point* that provides services to the other nodes, *e.g.*, WIFI or mesh network provider. Note that although in the Figure 1 there is only one access node, it is possible that there exist multiple access nodes. A *service requestor* node (shaded circle) is a regular terminal that would like to be provided with a specific service. It communicates with the access point in a multi-hop fashion via *intermediate terminals*.

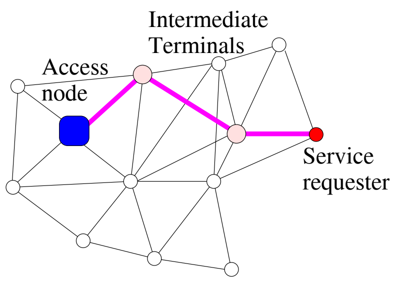


Figure 1: Network players

In this paper, we assume that each wireless terminal is not able to adjust their transmission power. It simplifies our model and enables us to focus on the non-cooperative issue. Note that our results are critical and can be borrowed to deal with the adjustable transmission power scenario. With the fixed transmission power, each terminal v_i incurs a cost of $h \cdot c_i$ to transmit a data packet of size h to its neighbors. Note that the cost c_i not only can be the deterministic cost under the binary link model where a packet is always received if the transmission power is above a threshold, but also can be the expected cost under the more realistic model where a packet is received with a probability [7, 24]. We assume that there is an central authority who collects payments from service requestors and distributes payments to the forwarding intermediate terminals. The service requestor's properties with respect to how much it is willing to spend (*i.e.*, its private type) can be captured by two models:

- **Axiom Model (AM).** The service requestor must receive the service, or equivalently, the service requestor has an infinitely large budget constraint for this service. In this model, we use $\eta^{=\infty}$ to denote that the budget constraint of the service requestor is infinity.
- **Valuation Model (VM).** The requestor t has a budget constraint w indicating the maximum value that t is willing to pay. In this model, t is selfish and can declare a budget constraint η that may not equal its actual budget constraint w.

Eidenbenz *et al.* [8] proposed a unicast system to charge the service requestor an amount that equals the cost of the second shortest path (*i.e.*, global disjoint replacement path) and pay the relay terminals according to the VCG mechanism. Their unicast system is strategyproof for both the service requestor and the relay terminals. However, solely achieving strategyproofness for both the relay terminals and service requestor is trivial: we can charge the service requestor zero or some arbitrarily high amount and pay the relay terminals according to VCG mechanism. The main drawback of the unicast systems we mentioned is that the central authority either gets nothing or too much from unicast routing. Thus, budget-imbalance issue should be taken into account through the notion of (α, β)-budget-balanced: A mechanism is (α, β)-budget-balanced if the central authority (1) can retrieve at least a fraction α of the total payment from the service requestor, and (2) can never retrieve more than β times of the total payments. In literature, either α or β is assumed to be 1. For all results presented in this paper, we assume that $\beta = 1$ and simply call $(\alpha, 1)$-budget-balanced as α-**budget-balanced**. All our results for α-budget-balanced can be easily extended to (α, β)-budget-balanced by replacing α in our formulas with $\frac{\alpha}{\beta}$. Besides the strategyproofness and budget-balance, another important property often required for a mechanism is *social efficiency*. A mechanism is ϱ-social-efficient if the output of the mechanism has cost at most ϱ times of the optimum. If $\varrho = 1$, then the mechanism is simply called social efficient. For example, a unicast mechanism is *social efficient* if it always uses the path with the least *true* cost (not necessarily the least *declared* cost by relay agents). It is well-known that no mechanism can be strategyproof, social efficient and budget-balanced simultaneously.

We call a mechanism system α-**perfect** if it (I) is strategyproof for both the relay terminals and the service requestor; (II) is α-budget-balanced; (III) satisfies some other requirements (namely NPT, CS) that will be discussed in detail later. In this paper, we prove that $\alpha \leq \frac{1}{n}$ for any unicast system that is α-perfect, where n is the number of terminals in the network (Section 4); the proof is based on an intuitive counter-example. We then present a unicast system that is $\frac{1}{2n}$-perfect, which is asymptotically optimal (also Section 4). The mechanisms for this system relies on the novel graph-theoretic concept of *Least Bridge Covers*.

If a unicast system is α-perfect, then the central authority can run into a deficit of up to a fraction of $1 - \alpha$ of the total payments to the relay terminals. From the negative results we show, the central authority can not avoid suffering large balance lost if they insist on that unicast system should be strategyproof for the relay terminals. In order to further reduce this budget imbalance, we propose to relax the strategyproof requirement for the relay terminals. Concretely, we propose to to design mechanisms that use Nash Equilibrium solutions instead of (weakly) dominant strategy solutions. We propose the LCPA mechanism with this property in the axiom model (Section 3). Note that Nash Equilibrium notation has a major drawback compared to strategyproof mechanism because it is very hard to make the system converge to a certain Nash Equilibrium in a *distributed* setting. It is indeed that LCPA mechanism can induce infinite number of different Nash Equilibria and we are not able to find the Nash Equilibrium with minimum payment even in a *centralized* manner. However, under our LCPA mechanism, we proved that the payment of *any* Nash Equilibrium is at most 2 times the minimum. This showed that in real implementation, we do not need to care much about to which Nash Equilibrium the system converges. Based on LCPA mechanism, we design a system that is budget-balanced with ϵ additive for any fixed positive ϵ under the valuation model (Section 4.2).

It is generally acknowledged that the unicast system based on strategyproof solution is very steady while we show in this paper that the unicast system based on LCPA mechanism achieves constant budget balance with arbitrary small additive. Thus, both the unicast systems could be used in practice according to different requirements of the application. It is worth to point out that our main intention in this paper is not to show which system is better. Our main focus is to design optimal unicast systems under both solutions, which could be used in wireless mesh networks, wireless hot-spots and more. To the best of our knowledge, our work is the *first* one that designs the optimal unicast systems based on strategyproof or Nash Equilibrium solutions. Therefore, we call the proposed systems: *O*ptimal *U*nicast *R*outing *S*ystems (OURS).

We conclude by discussing the details of the efficient implementation for different application needs (Section 5), presenting simulation results that validate our claims for efficiency and budget imbalance in realistic scenarios (Section 6), and by summarizing remarks (Section 7).

2. SCENARIO DESCRIPTION

We need to recall a few definitions and concepts from mechanism design. A standard model for mechanism design is as follows. There are n agents $1, \ldots, n$. Each agent i has some private information t_i, called its *type*, only known to itself. For example, the type t_i can be the cost that agent i incurs for forwarding a packet in a network or can be the maximum payment that the agent i is willing to pay for a service as a service requestor. The agents' types define the *type vector* $t = (t_1, t_2, \ldots, t_n)$. Each agent i has a set of strategies A_i from which it can choose. For each strategy vector $a = (a_1, \ldots, a_n)$ where agent i plays strategy $a_i \in A_i$, the *mechanism* $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ computes an *output* $o = \mathcal{O}(a)$ and a *payment* vector $\mathcal{P}(a) = (\mathcal{P}_1(a), \ldots, \mathcal{P}_n(a))$. Here the payment $\mathcal{P}_i(\cdot)$ is the money given to agent i and depends on the strategies used by the agents. A *valuation* function $v_i(o)$ assigns a monetary amount to agent i for each possible output o. Let $u_i(o)$ denote the *utility* of agent i at the output o of the game. Following a common assumption in the literature, we assume that the utility for agent i is quasi-linear, *i.e.*, $u_i(o) = v_i(o) + \mathcal{P}_i(a)$. We adopt the assumption in neoclassic economics that every agent will optimize its utility.

A strategy vector $\mathbf{a}$ is called a *Nash Equilibrium* if a_i maximizes the utility of agent i when the strategies of all the other agents are fixed as $\mathbf{a}_{-i}$, where $\mathbf{a}_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$ denotes the strategies of all the other agents except i. A strategy a_i is (weakly) *dominant* for agent i if it maximizes its utility regardless of possible strategies of the other agents, *i.e.*, $u_i(t_i, \mathcal{O}(\mathbf{a}_i)) \geq u_i(t_i, \mathcal{O}(a'_i, \mathbf{a}_{-i}))$ for all $a'_i \neq a_i$ and all strategies $\mathbf{a}_{-i}$.

A *direct-revelation* mechanism is a mechanism in which the only actions available to each agent are to report its private type either truthfully or falsely to the mechanism. A direct-revelation mechanism is *incentive compatible* (IC) if reporting valuation truthfully is a dominant strategy. Another very common requirement in the literature for mechanism design is so called *individual rationality* (IR): the agent's utility of participating in the output of the mechanism is not less than the utility of the agent if it did not participate at all. A direct-revelation mechanism is called *truthful* or *strategyproof* if it satisfies both IC and IR properties. Notice that strategyproof mechanism always pays a certain amount that is not less than the actual cost to induce the truthfulness. The difference between the total amount paid and the total cost the agents should spend are usually called the *premium* or *overpayment*.

The generalized VCG mechanisms by Vickrey [21], Clarke [6], and Groves [10] may be arguably the most important positive result in mechanism design. An objective function $g(o, t)$ is called *utilitarian* if $g(o, t) = \sum_i v(t_i, o)$. The VCG mechanisms apply to (affine) maximization problems where the objective function is utilitarian and the set of possible outputs is finite. A direct-revelation mechanism $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ belongs to the VCG family if (1) the output $\mathcal{O}(t)$ computed based on the type vector t maximizes the utilitarian objective function, and (2) the payment to agent i is $\mathcal{P}_i(t) = \sum_{j \neq i} v_j(t_j, \mathcal{O}(t)) + h^i(t_{-i})$. Here $h^i(\cdot)$ is an arbitrary function of t_{-i}. Green and Laffont [9] proved that, under mild assumptions, the VCG mechanisms are the only truthful mechanism for utilitarian maximization problems.

We are now ready to describe our scenario in more detail and propose new definitions. Formally, we assume that there is a network $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of the wireless terminals and $E = \{e_1, e_2, \ldots, e_m\}$ is the wireless communication links in which $e_k = v_i v_j$ means that terminal v_i and v_j can communicate with each other directly. Without loss of generality, we assume that s is the source, t is the service requestor and s and t are in V. We also assume that there is a fixed cost c_i for wireless terminal v_i to transmit a unit size data and the cost to transmit a traffic of size h is $h * c_i$. Let $\mathbf{c}$ be the cost vector of the terminals, *i.e.*, $\mathbf{c} = (c_1, c_2, \ldots, c_n)$. Every wireless terminal v_i is required to declare a cost, say d_i, for forwarding the unit size data. The set of all terminals' declared cost is denoted as $\mathbf{d} = (d_1, \ldots, d_n)$. Notice that d_i may not equal c_i, which is v_i's actual cost. For simplicity of our analysis, we normalize the traffic to unit size data. Dropping this assumption does not change the results. Our aim is to design *unicast systems*.

DEFINITION 1. *A* unicast system *(US)* $\Psi = (\mathcal{M}, \mathcal{S})$ *consists of a routing mechanism* $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ *and a charging mechanism* $\mathcal{S} = (\sigma, \xi)$*. Let* $\mathbf{d}$ *be the vector of declarations by all relay agents and* η *be the declared maximum willing payment by the requestor.* $\mathcal{O}(\eta, \mathbf{d}) = (\mathcal{O}_1(\eta, \mathbf{d}), \ldots, \mathcal{O}_n(\eta, \mathbf{d}))$ *is an output function vector such that* $\mathcal{O}_i(\eta, \mathbf{d})$ *indicates the times terminal* v_i *should send a packet, i.e.,* $\mathcal{O}_i(\eta, \mathbf{d}) = 1$ *indicates that terminal* v_i *should send the packet once.* $\mathcal{P}(\eta, \mathbf{d}) = (\mathcal{P}_1(\eta, \mathbf{d}), \ldots, \mathcal{P}_n(\eta, \mathbf{d}))$ *is a payment function vector that computes the payment for the terminals, i.e.,* $\mathcal{P}_i(\eta, \mathbf{d})$ *is payment to terminal* v_i*.* $\sigma(\eta, \mathbf{d})$ *is an indication function for service requestor* t*, i.e.,* $\sigma(\eta, \mathbf{d}) = 1$ *or* 0 *indicates whether the service requestor* t *can receive the data or not.* $\xi(\eta, \mathbf{d})$ *is a charging function for the service requestor* t*, i.e.,* $\xi(\eta, \mathbf{d})$ *computes how much the service requestor* t *should be charged for the data transmission. If the requestor* t *can receive the data, then the data must be routed along the least cost path between access point* s *and requestor* t*, i.e., our unicast system is social efficient.*

Let $\mathbb{P}(\eta, \mathbf{d})$ be total payment to the terminals, *i.e.*, $\mathbb{P}(\eta, \mathbf{d}) = \sum_{v_i} \mathcal{P}(\eta, \mathbf{d})$. For notational simplicity, we also use $\mathcal{O}(\eta, \mathbf{d})$ to denote the terminal set that is selected to route the data. Under Axiom Model (AM), the service requestor always receives the service and pays the total payment. Thus, the focus is on the routing mechanism $\mathcal{M} = (\mathcal{O}, \mathcal{P})$ in that case. But ideally, a unicast system should satisfy some requirements under Valuation Model (VM).

DEFINITION 2. *Under the VM, a unicast system* $\Psi = (\mathcal{M}, \mathcal{S})$ *is* ***perfect*** *if it satisfies that*

1. ***Strategyproof:*** $\mathcal{M}$ *is strategyproof for relay terminals, and* $\mathcal{S}$ *is strategyproof for the service requestor* t*. In other words, every terminal declares* $d_i = c_i$ *and requestor* t *declares* $\eta = w$ *to maximize their utility.*
2. ***Budget Balance*** *(BB): System* Ψ *satisfies* ***Cost Recovery*** *if* $\xi(\eta, \mathbf{c}) \geq \mathbb{P}(\eta, \mathbf{c})$*, i.e., the total payment is recovered from*

the service requestor. Ψ *satisfies* ***Competitiveness*** *if* $\xi(\eta, \mathbf{c}) \leq \mathbb{P}(\eta, \mathbf{c})$*, i.e., no surplus is created. Because if any surplus is created then a competitor who provides the same service may come at a cheaper price by reducing surplus.*
Ψ *is* budget balanced *if both cost recovery and competitiveness are satisfied. When budget-balance cannot be met, we relax it to* α*-budget-balanced:* $\alpha \cdot \mathbb{P}(\eta, \mathbf{d}) \leq \xi(\eta, \mathbf{d}) \leq \mathbb{P}(\eta, \mathbf{d})$*, for some given parameter* $\alpha \in [0, 1]$*, which is called the* budget balance factor *(BBF).*

3. ***No Positive Transfer*** *(NPT): The charge for service requestor* t *should not be negative. In other words, we don't pay the service requestor to receive the data.*
4. ***Consumer Sovereignty*** *(CS): The charging mechanism cannot arbitrarily exclude a service requestor; the requestor will get the service if* η *is sufficiently large while others are fixed.*

Since our unicast system is always social efficient, obviously we cannot have a perfect unicast system. As a variation, we say that, under the VM, a unicast system $\Psi = (\mathcal{M}, \mathcal{S})$ is α-**perfect** if it satisfies NPT, CS, strategyproof and α-budget-balance for $\alpha \in [0, 1)$.

Instead of insisting on strategyproofness, we may design some mechanisms that use solution of Nash equilibria, based on the following definitions.

DEFINITION 3. *A unicast system* Ψ *is* α*-NE-budget-balanced if* $\alpha \cdot \mathbb{P}(\eta, \widetilde{\mathbf{b}}) \leq \xi(\eta, \widetilde{\mathbf{b}}) \leq \mathbb{P}(\eta, \widetilde{\mathbf{b}})$ *holds for any bid vector* $\widetilde{\mathbf{b}}$ *that is a NE for* Ψ*, where* $\alpha \leq 1$*. Similarly,* Ψ *satisfies NE-CS if for any fixed NE* $\widetilde{\mathbf{b}}$ *of* Ψ*, there exists a valuation* x *for* t *such that* $\sigma(\eta, \widetilde{\mathbf{b}}) = 1$ *for any* $\eta > x$*.* Ψ *is* α***-NE-perfect*** *if (1)* Ψ *satisfies NPT, NE-CS, (2) it is strategyproof for the service requestor, and (3) it is* α*-NE-budget-balanced.*

Notations. Following we introduce some terminologies and symbols that will be used later. For a simple path $\mathsf{P} = v_i \rightsquigarrow v_j$, we define the weight of the path under cost vector $\mathbf{d}$ as $\omega(\mathsf{P}, \mathbf{d}) = \sum_{v_k \in \mathsf{P} - v_i - v_j} d_k$. The path between two terminals s, t under declared cost vector $\mathbf{d}$ with minimal weight is denoted as $\mathsf{LCP}(s, t, \mathbf{d})$, which stands for *lease cost path*. Among all paths between s, t with terminal v_k on it, the shortest path is denoted as $\mathsf{LCP}_{v_k}(s, t, \mathbf{d})$. Similarly, among all paths between s, t without terminal v_k on it, the shortest path is denoted as $\mathsf{LCP}_{-v_k}(s, t, \mathbf{d})$.

Let $s(\mathsf{P})$ and $t(\mathsf{P})$ be two end terminals of P. If $s(\mathsf{P})$ and $t(\mathsf{P})$ are the only two terminals that is also on $\mathsf{LCP}(s, t, \mathbf{d})$, then P is a *bridge* over $\mathsf{LCP}(v_i, v_j, \mathbf{d})$. P *covers* v_k if it is an internal node of $\mathsf{LCP}(v_i, v_j, \mathbf{d})$. Without loss of generality, we also assume that $s(\mathsf{P})$ is the terminal closer to s than $t(\mathsf{P})$.

We summarize the notations used in the following table.

V	The set of wireless terminals in the network
E	The set of wireless communication links
$\mathbf{c}$	The actual cost vector of terminals
$\mathbf{d}$	Declared cost vector of terminals
$\omega(\mathsf{P}, \mathbf{d})$	The weight of path P under cost vector $\mathbf{d}$
$\mathsf{LCP}(s, t, \mathbf{d})$	Least cost path between s and t under $\mathbf{d}$
w	Actual budget constraint of service requestor t
η	Declared budget of service requestor t
$\mathcal{O}$	Output function selects the relay terminals
$\mathcal{P}$	Payment function computes the payment
$\mathbb{P}$	Total payment to the relay terminals
$\mathcal{M} = (\mathcal{O}, \mathcal{P})$	Routing mechanism composed of $\mathcal{O}$ and $\mathcal{P}$
σ	Method decides whether t receives the service
ξ	Method computes how much t is charged
$\mathcal{S} = (\sigma, \xi)$	Charging scheme composed of σ and ξ
$\Psi = (\mathcal{M}, \mathcal{S})$	Unicast system with a routing mechanism $\mathcal{M}$ and a charging scheme $\mathcal{S}$

3. UNICAST SYSTEM UNDER AM MODEL

3.1 Dominant Strategy - VCG Revisited

Unicast routing game has been studied extensively since it is introduced by Nisan and Ronen [18]. They solved the unicast routing game by applying the VCG mechanism in a centralized way. Let $\mathcal{P}^{\mathsf{VCG}}$ denote the VCG payment for unicast mechanism under AM, and $\mathbf{d}|^k d'_k = (d_1, \ldots, d_{k-1}, d'_k, d_{k+1}, \ldots, d_m)$. The payment to terminal $v_k \in \mathsf{LCP}(s, t, \mathbf{d})$ according to VCG mechanism is

$$\mathcal{P}_k^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d}) = |\mathsf{LCP}(s, t, \mathbf{d}|^k \infty)| - |\mathsf{LCP}(s, t, \mathbf{d}|^k 0)|,$$

where $\eta^{=\infty}$ means that the service requestor has an infinity valuation. Following is a simple property of the VCG mechanism.

FACT 1. *For any* $v_i \in \mathsf{LCP}(s, t, \mathbf{d})$*,* $d_i \leq \mathcal{P}_k^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d})$*.*

VCG mechanism based unicast system is social efficient, strategyproof, but its budget balance factor could be as small as $\frac{1}{n}$: the total payment to relay agents could be n times of their actual costs.

3.2 Nash Equilibrium for Terminals

In light of the potential large overpayment of the VCG mechanism, which is the *only* strategyproof mechanism based on the LCP, it is natural to relax the dominant strategy to the Nash equilibrium, which is a weaker requirement. In this section, we design a new mechanism, which we call *Least Cost Path Auction* (LCPA) mechanism, that can induce some Nash Equilibria under AM model.

Usually, a naive mechanism coming into one's mind is that each terminal declares a single bid, and we select the LCP based on the bids and pay the terminals on the path whatever they declared. However, the naive mechanism suffers from two major problems. Consider a network shown in Figure 2 where the number in the bracket shows the actual costs of the terminals. We assume that sv_3t is always selected if the weight of the two paths are the same (called *tie break* afterward). When v_1 declares x, v_2 and v_3 both declare $x - 1$ where x is any value greater than 9, the system reaches a NE. Thus, the system could reach some NE that need arbitrary large overpayment compared with VCG payment. The root cause of this problem is that the terminals who are not selected do not have incentive to declare their true costs. Even we can ensure that the terminals who are not selected always declare their true costs, the existence of the NE is not guaranteed. Consider the network in Figure 2 again and terminal v_3 declares its true cost 9. If there exits a NE, since sv_3t always wins over sv_1v_2t when there is a tie break, the sum of the declared costs of v_1 and v_2 must be smaller than 9. On the other hand, if the total declared costs of v_1 and v_2 is $9 - \epsilon$ for any ϵ, either v_1 or v_2 can increase its declared cost by $\frac{\epsilon}{2}$ to increase their payment. Thus, there does not exist any NE.

The LCPA mechanism solves the above two problems by requiring two bids instead of one from each terminal. The LCPA mechanism is divided into two phases: broadcast phase and unicast phase. In the broadcast phase, each terminal sends a"dummy" packet and receives a small payment such that its utility is maximized *if and only if* it bids its true cost *no matter* what other terminals do. Since those terminals are not selected finally only receive payment in broadcast phase, they always declare their true costs, which enables LCPA mechanism to remedy the first problem of the naive system. More detailed discussion of what the dummy packets contain and how often it should be sent is discussed in Section 5.1. In unicast phase, we first choose a *candidate LCP* $\mathsf{LCP}(s, t, \mathbf{b})$ using the first bid vector $\mathbf{b}$. A bid vector $\mathbf{h}$ is composed of the second bids from terminals on candidate LCP and first bids from terminals not on candidate LCP, *i.e.*, we only use the second bids from the terminals that are on candidate LCP. We choose the path $\mathsf{LCP}(s, t, \mathbf{h})$

using bid vector $\mathbf{h}$ as the final path to relay the packets. Only the terminals on $\mathsf{LCP}(s,t,\mathbf{h})$ get a payment h_i and all terminals originally on candidate path but not on final path should receive a fine $\gamma\cdot|b_i-b'_i|$ for bidding too greedy. Algorithm 1 presents our LCPA mechanism.

To show how LCPA mechanism Ψ^{LCPA} works, we give two example scenarios shown in Figure 2. The number in the bracket shows the actual costs of the terminals, and two numbers in <> are the bids. In scenario (1), $\mathsf{LCP}(s,t,\mathbf{b})$ is sv_1v_2t. Thus, $h_1 = b'_1 = 5$, $h_2 = b'_2 = 4$ and $h_3 = b_3 = 9$. Note that under cost vector $\mathbf{h}$, both paths sv_1v_2 and sv_3 have the same cost 9. However, according to the tie-breaking rule, we will choose sv_1v_2 because it contains more nodes from $\mathsf{LCP}(s,t,\mathbf{b})$. Note that this tie-breaking rule is critical to our LCPA mechanism. Thus, v_1 gets 5 and v_2 gets 4 in the unicast phase. In scenario (2), the LCP chosen based on $\mathbf{b}$ is also sv_1v_2t. Similarly, $h_1 = b'_1 = 5$, $h_2 = b'_2 = 5$ and $h_3 = b_3 = 9$. However, path sv_1v_2t has cost 10 in the cost vector $\mathbf{h}$, which is greater than the cost of path sv_3t. Thus, v_1 and v_2 are punished by fine -3γ and -2γ respectively. Terminal v_3 gets a payment 9 in the unicast phase and the actual routing path is sv_3t.

Regarding the broadcast phase in Algorithm 1, we have the following lemma.

LEMMA 1. *For each terminal v_i, the utility for broadcast $g_i(\mathbf{b}) = -\rho\cdot c_i + f_i(s,t,\mathbf{b})$, strictly decreases in $[c_i,+\infty)$ and strictly increases in $(-\infty,c_i]$ on b_i.*

PROOF. Simplifying $g_i(\mathbf{b})$ obtains that $g_i(\mathbf{b}) = -\rho\cdot c_i + f_i(s,t,\mathbf{b}) = \tau_i(b_{-i})\left[(b_u-c_i)\cdot(n\cdot b_u - b_{-v_i}) + \frac{c_i^2}{2} - \frac{(c_i-b_i)^2}{2}\right]$, where $b_{-v_i} = \sum_{v_j\in G-v_i} b_j$. Thus $g_i(\mathbf{b})$ is a function on b_i that decreases in $[c_i,+\infty)$ and increases in $(-\infty,c_i]$. □

Algorithm 1 Mechanism $\mathcal{M}^{\mathsf{LCPA}} = (\mathcal{O}^{\mathsf{LCPA}},\mathcal{P}^{\mathsf{LCPA}})$.

Input: Network $G=(V,E)$, a source s, a service requestor t, declared cost vector $\widetilde{\mathbf{b}} = \langle\mathbf{b},\mathbf{b}'\rangle$, and a parameter γ.
Output: $\mathcal{M}^{\mathsf{LCPA}}$.

1: Set $\mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) = f_i(s,t,\mathbf{b})$, where $f_i(s,t,\mathbf{b}) = \tau_i(b_{-i})\cdot\left[b_u\cdot(n\cdot b_u - \sum_{v_j\in G-v_i} b_j) - \frac{b_i^2}{2}\right]$, for each terminal $v_i\in G$. Here, b_u is the maximum cost that any terminal can declare and $\tau_i(b_{-i})$ is *any* function that does not depend on b_i.
2: Each terminal sends a "dummy" packet of size $\rho = \tau_i(b_{-i})\cdot(n\cdot b_u - \sum_{v_i\in G} b_i)$.
3: Compute the path $\widetilde{\mathsf{P}} = \mathsf{LCP}(s,t,\mathbf{b})$, which we call *candidate LCP*, and for each terminal v_i on $\widetilde{\mathsf{P}}$, set $h_i = b'_i$, set $h_i = b_i$ for other terminals.
Remark: Previous is the unicast phase and following is the unicast phase.
4: Compute path $\mathsf{LCP}(s,t,\mathbf{h})$, which is the *final LCP*, and break ties according to the following rule: if two paths have the same cost, then we choose the one that contains more terminals from candidate LCP $\mathsf{LCP}(s,t,\mathbf{b})$.
5: Set $\mathcal{O}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) = 1$ and $\mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) + h_i$ for each terminal on $\mathsf{LCP}(s,t,\mathbf{h})$.
6: Set $\mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty},\widetilde{\mathbf{b}}) - \gamma\cdot|b'_i - b_i|$ for each terminal on $\mathsf{LCP}(s,t,\mathbf{b}) - \mathsf{LCP}(s,t,\mathbf{h})$.

Recall that for naive mechanism, there may not exist some NE. Following Theorem shows that there must exist some NE under LCPA mechanism.

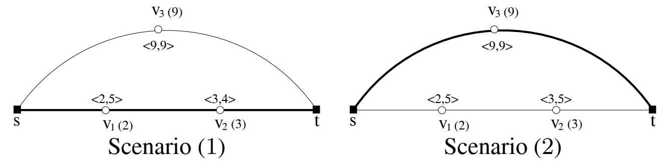


Figure 2: Illustration of LCPA mechanism.

THEOREM 2. *There exists Nash Equilibrium for LCPA Mechanism $\mathcal{M}^{\mathsf{LCPA}}$.*

PROOF. We prove by explicitly constructing a bid vector that is a NE for $\mathcal{M}^{\mathsf{LCPA}}$. Without loss of generality, assume that $\mathsf{P} = \mathsf{LCP}(s,t,\mathbf{c}) = sv_1v_2\ldots v_kt$. We initialize the cost vector $\mathbf{c}^{(0)} = \mathbf{c}$ and iteratively process the terminals from v_1 to v_k as follows: compute VCG payment $\mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{c}^{(i-1)})$ for v_i using the cost vector $\mathbf{c}^{(i-1)}$, and obtain the new cost vector $\mathbf{c}^{(i)}$ from $\mathbf{c}^{(i-1)}$ by setting $c_i^{(i)} = \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{c}^{(i-1)})$. Let $\mathbf{c}' = \mathbf{c}^{(k)}$ and the bidding vector $\widetilde{\mathbf{b}} = \langle\mathbf{c},\mathbf{c}'\rangle$. We prove that the bidding vector $\widetilde{\mathbf{b}} = \langle\mathbf{c},\mathbf{c}'\rangle$ is a NE by contradiction. For the sake of contradiction, we assume that terminal v_i can increase its utility by declaring a bid pair $\langle x,y\rangle$ that is different from $\langle c_i,c'_i\rangle$. We discuss by cases.

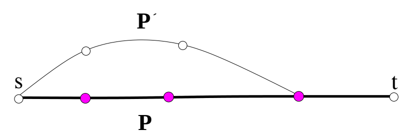


Figure 3: The illustration of the path P and P'.

Case 1: Terminal $v_i\in\mathsf{LCP}(s,t,\mathbf{c})$. There are two subcase here. (1) If $v_i\in\mathsf{LCP}(s,t,\mathbf{c}|^i x)$, *i.e.*, v_i is in the candidate LCP, then the utility in the broadcast phase is $g_i(\mathbf{c}|^i x)\le g_i(\mathbf{c})$ from Lemma 1. In the unicast phase, c'_i is the maximum value v_i can declare when it is still in $\mathsf{LCP}(s,t,\mathbf{c}')$. Thus, the utility of v_i does not increase. (2) If $v_i\notin\mathsf{LCP}(s,t,\mathbf{c}|^i x)$, *i.e.*, v_i is not in the candidate LCP anymore, then $x>c_i$. Let $\mathsf{P}'\in\mathsf{LCP}(s,t,\mathbf{c}|^i x)$ that covers v_i and $\mathsf{P}\in\mathsf{LCP}(s,t,\mathbf{c})$ as shown in Figure 3. Note that for any terminal $v_j\notin\mathsf{LCP}(s,t,\mathbf{c})$, $c'_j = c_j$. Thus, for any terminal $v_j\in\mathsf{P}'$, $h_i = c'_i = c_i$. On the other hand, since path P is not of part of the candidate LCP, $h_j = c_j$ for any terminal $v_j\in\mathsf{P}-v_i$. From the assumption P' is on the candidate LCP, $\omega(\mathsf{P}',\mathbf{c})\le\omega(\mathsf{P},\mathbf{c}|^i x)$. Thus,

$$\begin{aligned}\omega(\mathsf{P}',\mathbf{h}) &= \sum_{v_j\in\mathsf{P}'} h_j = \sum_{v_j\in\mathsf{P}'} c_j = \omega(\mathsf{P}',\mathbf{c}) \le \omega(\mathsf{P},\mathbf{c}|^i x)\\ &= \sum_{v_j\in\mathsf{P}-v_i} c_j + x = \sum_{v_j\in\mathsf{P}-v_i} h_j + x = \omega(\mathsf{P},\mathbf{h}|^i x),\end{aligned}$$

which implies that v_i is not selected in the unicast phase. Therefore, its overall utility is $g_i(\mathbf{c}|^i x)$. Now we conclude that v_i can not increase it utility by declaring a bid pair $\langle x,y\rangle$ that is different from $\langle c_i,c'_i\rangle$ in this case.

Case 2: Terminal $v_i\notin\mathsf{LCP}(s,t,\mathbf{c})$. There are also two subcases here. (1) If terminal $v_i\notin\mathsf{LCP}(s,t,\mathbf{c}|^i x)$, *i.e.*, v_i is neither on the candidate LCP, then $\mathbf{h} = \mathbf{c}'|^i x$. v_i's utility from broadcast phase is $g_i(\mathbf{c}|^i x)$, which is not greater than $g_i(\mathbf{c})$ from Lemma 1. If terminal v_i is not on $\mathsf{LCP}(s,t,\mathbf{h})$, then its overall utility is $g_i(\mathbf{c}|^i x)$, which is smaller than $g_i(\mathbf{c})$. If terminal v_i is on $\mathsf{LCP}(s,t,\mathbf{h})$, then $x<c_i$, which means that v_i's utility in the unicast phase is $x-c_i<0$. Thus, terminal v_i does not increase its overall utility. (2) If terminal $v_i\in\mathsf{LCP}(s,t,\mathbf{c}|^i x)$, *i.e.*, v_i manage to decrease its first bid x in order to be on candidate LCP, then $x < c_i$. Thus, the utility in the broadcast phase decreases. Let $\mathsf{P} \in \mathsf{LCP}(s,t,\mathbf{c})$ be a bridge that covers $\mathsf{P}' \in \mathsf{LCP}(s,t,\mathbf{c}|^i x)$ and $v_i \in \mathsf{P}'$. If $v_i \notin \mathsf{LCP}(s,t,\mathbf{h})$, then it has utility $-\gamma \cdot |b_i' - b_i|$ in the unicast phase. If $v_i \in \mathsf{LCP}(s,t,\mathbf{h})$, then $\omega(\mathsf{P}',\mathbf{h}) \le \omega(\mathsf{P},\mathbf{h})$. Similarly to the arguement of case 1, each terminal v_j in P and $\mathsf{P}' - v_i$ has cost c_j. Thus, y must be smaller than c_i. Therefore, v_i's utility in the unicast phase is $y - c_i < 0$. This shows that v_i's overall the utility decreases when v_i bids $\langle x, y\rangle$ instead of $\langle c_i, c_i'\rangle$ in this case. This finishes our proof. □

More generally, if $\widetilde{\mathbf{b}}$ is any Nash Equilibrium for LCPA mechanism $\mathcal{M}^{\mathsf{LCPA}}$, we have the following lemma (its proof is presented in the appendix).

LEMMA 3. *Assume that $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}'\rangle$ is a Nash Equilibrium for LCPA mechanism $\mathcal{M}^{LCPA}$, where $\mathbf{h}$ is the cost vector obtained in Algorithm 1.*

1. *$\mathbf{b} = \mathbf{c}$, i.e., each terminal declares its true cost as the first bid.*

2. *$LCP(s,t,\mathbf{c}) = LCP(s,t,\mathbf{h})$, i.e., LCPA always chooses actual LCP.*

3. *For any $v_i \in LCP(s,t,\mathbf{b}) = LCP(s,t,\mathbf{c})$, $|LCP(s,t,\mathbf{h})| = |LCP_{-v_i}(s,t,\mathbf{h})|$.*

Unlike the VCG mechanism that always has the same total payment for a fixed cost vector $\mathbf{c}$, the total payments may varies under different NEs for LCPA mechanism. Let $\widetilde{\mathbf{b}}^{\min}$ and $\widetilde{\mathbf{b}}^{\max}$ be any two NEs of the LCPA mechanism such that the total payment is minimized and maximized respectively. Usually, person favor truthful mechanism over the Nash Equilibrium because the system may have multiple Nash Equilibria and it is almost impossible to reach some specific Nash Equilibria in a distributed setting. However, if the system performance under different Nash Equilibrium differs not much, then we are not so worried about which Nash Equilibrium the system converges to. Fortunately, our LCPA mechanism does have this nice property and following theorem shows that the total payment at different NE differs at most 2 times.

THEOREM 4. $\mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\max}) \le 2 \cdot \mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\min})$.

PROOF. Karlin *et al.* [15] showed that

$$2 \cdot \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {b_j'}^{\min} \ge \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {b_j'}^{\max}.$$

From Statement 1 of Lemma 3, the total payment for the unicast phase is a function of $\mathbf{b} = \mathbf{c}$. Thus, as long as the true cost vector $\mathbf{c}$ is fixed, the total payment for unicast phase is fixed, say ϵ. From Statement 2 of Lemma 3, the total payment of unicast is $\sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} b_j'$ for any Nash Equilibrium $\widetilde{\mathbf{b}}$. Thus, $\mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\max}) = \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {b_j'}^{\max} + \epsilon \le 2 \cdot \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {b_j'}^{\min} + 2\epsilon = 2 \cdot \mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\min})$. □

There are vast literatures discuss how to measure the overpayment of a mechanism, and the notation of *frugality* [15] has been shown accurate and useful. The frugality of a mechanism is defined as the total payment over $\nu(\mathbf{c})$, where $\nu(\mathbf{c}) = \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {b_j'}^{\min}$. It has been shown by Karlin *et al.* that VCG mechanism has frugality $O(n)$ in unicast routing where n is the number of nodes. Contrary, our LCPA mechanism has a frugality $2+\epsilon$ for any given positive ϵ, which *greatly* improves the VCG mechanism and is also asymptotically optimal. Before presenting our theorem, we give a lemma that relate the $\nu(\mathbf{c})$ to the cost of least bridge cover $\mathbb{LB}(s,t,\mathbf{c})$.

LEMMA 5. *[Immorlica et al. [11] and Karlin et al. [15]] For any network, $\nu(\mathbf{c}) \le |\mathbb{LB}(s,t,\mathbf{c})| \le 2\nu(\mathbf{c})$.*

THEOREM 6. *For any given ϵ, by properly setting τ_i, the frugality of our LCPA mechanism is $2+\epsilon$, which is asymptotically optimal.*

PROOF. In the proof of Theorem 4, we obtain that $2\nu(\mathbf{c}) = 2 \cdot \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {\mathbf{b}'_j}^{\min} \ge \sum_{v_j \in \mathsf{LCP}(s,t,\mathbf{c})} {\mathbf{b}'_j}^{\max}$, which implies that total payment in the unicast phase is at most 2 times the $\nu(\mathbf{c})$.

In broadcast phase, by setting $\tau_i(b_{-i}) = \frac{\epsilon \cdot |\mathbb{LB}(s,t,\mathbf{b})|}{2n}$ for each $v_i \in \mathsf{LCP}(s,t,\mathbf{b})$ and $\tau_i(b_{-i}) = \frac{\epsilon \cdot |\mathsf{LCP}(s,t,\mathbf{b})|}{n}$ otherwise, the total payment in the broadcast phase is at most $\epsilon \cdot \nu(\mathbf{c})$ under any NE $\widetilde{\mathbf{b}}$. Thus, the total payment is at most $(2+\epsilon)\cdot\nu(\mathbf{c})$, which finishes our proof. □

Theorem 6 reveals an important fact: our LCPA mechanism could largely reduce the overpayment of the VCG mechanism in the worst case theoretically.

4. UNICAST SYSTEM UNDER VALUATION MODEL

In this section, we focus on how to design a unicast system that is α-perfect in the Valuation Model, and present some results on both the negative and positive side.

4.1 α-perfect Unicast System

4.1.1 α-perfect Unicast System– An Upper Bound

Recall that for a unicast system Ψ that is α-perfect, the central authority should pay $1-\alpha$ of the total payment. Thus, ideally, one may try to find the unicast system that is 1-perfect to balance the budget for the central authority. Naively, one may design a unicast system Ψ^N that is budget-balanced using VCG mechanism as follows. First choose the shortest path $\mathsf{LCP}(s,t,\mathbf{d})$ according to the declared vector $\mathbf{d}$. If the valuation of the service requester s is not smaller than $\mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$, then (1) s get the service and each node on the LCP relay the packet; (2) each relay node gets a VCG payment $\mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$. Otherwise, (1) s does not get the service and no nodes relay the packet; (2) each node gets a zero payment. Unfortunately, following example shows a surprising result: unicast system Ψ^N is not strategyproof for the relay nodes. Consider the same network in Figure 4. The valuation of s is ak, $c_i = 0$ for $1 \le i \le k$ and $c_{k+1} = ak$. If each node declares its true cost, then the VCG payment to each node on LCP is ak and the total payment is ak^2. Since the valuation of s is ak that is much smaller than ak^2 when $k > 1$, no nodes relay the packet and each node receives a zero payment. On the other hand, if node v_1 declares a cost ak, each node on LCP except v_1 gets a payment 0 and v_1 gets a payment ak. In this case, the total payment is ak and s receives the service. Consequently, v_1 relays the packet and get a payment ak. v_1's utility is ak when it falsely declares its cost as ak. This shows that unicast system Ψ^N is not strategyproof for the relay nodes. As a step further, following we show a very strong result on the negative side: we can not design α-perfect system for any α that is greater than $1/n$.

LEMMA 7. *Assume $\Psi = (\mathcal{M},\mathcal{S})$ is the unicast system. If Ψ is α-perfect, then **1.** There exists a function $\mu(\mathbf{d})$ such that (1) $\sigma(\eta,\mathbf{d}) = 1$ if and only if $\eta \ge \mu(\mathbf{d})$; (2) $\xi(\eta,\mathbf{d}) = \mu(\mathbf{d})$ if $\sigma(\eta,\mathbf{d}) = 1$ and $\xi(\eta,\mathbf{d}) = 0$ otherwise.*

2. *If $d_j < d_j' < \mathcal{P}_j^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$, then $\mu(\mathbf{d}) \le \mu(\mathbf{d}|^j d_j')$.*

PROOF. **Statement 1:** This statement follows directly from the previous results [14, 16, 17] and is omitted here.

Statement 2: We prove the second statement by contradiction. For the sake of contradiction, $\mu(\mathbf{d}) > \mu(\mathbf{d}|^j d'_j)$. Consider the case when service requestor t has valuation η such that $\mu(\mathbf{d}) > \eta > \mu(\mathbf{d}|^j d'_j)$ and $\mathbf{c} = \mathbf{d}$. If v_j reveals its true cost $d_j = c_j$, then service requestor t is not selected to receive the service. Thus, v_j is not selected and has utility 0. Consider the case when v_j declares it cost to d'_j. Since $\eta > \mu(\mathbf{d})$, t is selected to receive the service. Thus, v_j is selected and from IR, $\mathcal{P}_j(\eta, \mathbf{d}) \geq d'_j$. Therefore, v_j's utility $\mathcal{P}_j(\eta, \mathbf{d}) - d_j$ is greater than 0, which implies that $\mathcal{M}$ is not strategyproof. This finishes our proof. □

Based on Lemma 7, the following theorem reveals a negative result on unicast system Ψ that is α-perfect.

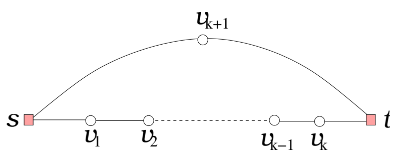


Figure 4: An example for unicast system.

THEOREM 8. *Assume that $\Psi = (\mathcal{M}, \mathcal{S})$ is unicast system that is α-perfect, then $\alpha \leq \frac{1}{n}$.*

PROOF. We prove it by presenting the network in Figure 4 as an example. Let $\mathbf{c}$ be the cost vector such that $c_i = 0$ for $1 \leq i \leq k$ and $c_{k+1} = ak$. Considering the cost vector $\mathbf{d}^{(1)} = \mathbf{c}|^1(ak - \epsilon)$. From Lemma 7,

$$\mu(\mathbf{c}) \leq \mu(\mathbf{d}^{(1)}). \quad (1)$$

Since each wireless terminal is either selected or not selected to relay, [2, 14, 16, 17] the strategyproof mechanism $\mathcal{M}$ should satisfy that for each terminal v_i, there is a threshold value $\kappa_i(\eta, \mathbf{d}_{-i})$ that does not depend on d_i such that (1) if $d_i < \kappa_i(\eta, \mathbf{d}_{-i})$ then $\mathcal{O}_i(\eta, \mathbf{d}) = 1$, (2) if $d_i > \kappa_i(\eta, \mathbf{d}_{-i})$ then $\mathcal{O}_i(\eta, \mathbf{d}) = 0$, and (3)$\mathcal{P}_i(\eta, \mathbf{d}) = \kappa_i(\eta, \mathbf{d}_{-i})$ if $\mathcal{O}_i(\eta, \mathbf{d}) = 1$ and zero otherwise. Since Ψ satisfies CS, there must exist a value $\eta \geq \mu(\mathbf{d}^{(1)})$ such that $\sigma(\eta, \mathbf{d}^{(1)}) = 1$. Following we proves by contradiction that $\kappa_i(\eta, \mathbf{d}^{(1)}_{-i}) \leq \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d}^{(1)})$. For the sake of contradiction, $\kappa_i(\eta, \mathbf{d}^{(1)}_{-i}) > \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d}^{(1)})$. Thus, there exists a small positive value δ such that when $d_i = \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d}^{(1)}) + \delta$, v_i is still on the shortest path $\mathsf{LCP}(s, t, \mathbf{d}^{(1)})$. This contradicts Fact 1. Thus, $\kappa_i(\eta, \mathbf{d}^{(1)}) \leq \epsilon$ for $2 \leq i \leq k$ and $\kappa_1(\eta, \mathbf{d}^{(1)}) \leq ak$. Therefore, $\mathbb{P}(\eta, \mathbf{d}^{(1)}) = \sum_{v_i} \mathcal{P}_i(\eta, \mathbf{d}^{(1)}) \leq ak + (k-1) \cdot \epsilon$. Since Ψ is α-budget-balance,

$$\mu(\mathbf{d}^{(1)}) \leq \mathbb{P}(\eta, \mathbf{d}^{(1)}) \leq ak + (k-1) \cdot \epsilon.$$

Combine Inequality (1) and above Inequality, we have

$$\mu(\mathbf{c}) \leq \mu(\mathbf{d}^{(1)}) \leq ak + (k-1) \cdot \epsilon.$$

Let cost vector $\mathbf{d}^{(i)}$ be $\mathbf{d}|^i(ak - \epsilon)$ for $1 \leq i \leq k$ and χ be a large positive number such that $\chi \geq \max_{1 \leq i \leq k} \xi(\mathbf{d}^{(i)})$. From the way we choose the valuation χ, the service requestor t receives the service under $\mathbf{d}^{(i)}$. Thus, v_i is selected and from IR, $\kappa_i(\chi, \mathbf{c}_{-i}) = \kappa_i(\chi, \mathbf{d}^{(i)}_{-i}) = \mathcal{P}_i(\chi, \mathbf{d}^{(i)}) \geq ak - \epsilon$. Consider the cost vector $\mathbf{c}$ and service requestor valuation χ. For any terminal v_i such that $1 \leq i \leq k, \mathcal{P}_i(\chi, \mathbf{c}) = \kappa_i(\chi, \mathbf{c}_{-i}) \geq ak - \epsilon$. Therefore, $\mathbb{P}(\chi, \mathbf{c}) = \sum_{v_i} \mathcal{P}_i(\chi, \mathbf{c}) \geq k \cdot (ak - \epsilon)$. Now we obtain

$$\alpha \leq \frac{\mu(\mathbf{c})}{\mathbb{P}(\chi, \mathbf{c})} \leq \frac{ak + (k-1) \cdot \epsilon}{k \cdot (ak - \epsilon)}.$$

Let $\epsilon \to 0$, $a \to \infty$ and $k = n$, then $\alpha \leq \frac{1}{n}$. This proves the theorem. □

Theorem 8 reveals an upper bound for the budget balance factor α on any unicast system Ψ that is α-perfect. Following, we present a unicast system that is $\frac{1}{2n}$-perfect, which is asymptotically optimal.

4.1.2 $\frac{1}{2n}$-perfect Unicast System

In [8], Eidenbenz proposed a unicast system that charge the service request an amount that equals to the weight of the *second shortest path*, where the second shortest path is the shortest path in graph $G - \mathsf{LCP}(s, t, \mathbf{d})$. However, the second shortest path could be arbitrary large than the total VCG payment. Thus, the unicast system presented in [8] is not α-perfect for any α because it violates the competitiveness. In order to remedy this, we proposed a unicast system that is based on the least bridge cover that is $1/2n$-perfect, which is asymptotically optimal. Our mechanisms are based on our prelimary results presented in [23].

Before presenting our unicast system, we first introduce some notations. A bridge set $\mathscr{B}$ is a *bridge cover* for $\mathsf{LCP}(s, t, \mathbf{d})$, if for every terminal $v_i \in \mathsf{LCP}(s, t, \mathbf{d})$, there exists a bridge $\mathrm{B} \in \mathscr{B}$ that covers v_i. The *weight* of a bridge cover $\mathscr{B}(s, t, \mathbf{d})$ is defined as $|\mathscr{B}(s, t, \mathbf{d})| = \sum_{B \in \mathscr{B}(s,t,\mathbf{d})} |B(\mathbf{d})|$. A bridge cover $\mathscr{B}$ is a *minimal bridge cover* (MBC), if for each bridge $B \in \mathscr{B}$, $\mathscr{B} - B$ is not a bridge cover. A bridge cover is a *least bridge cover* (LBC), denoted by $\mathbb{LB}(s, t, \mathbf{d})$, if it has the smallest weight among all bridge covers that cover $\mathsf{LCP}(s, t, \mathbf{d})$. It is easy to show that $\mathbb{LB}(s, t, \mathbf{d})$ could be computed in $O(n \log n) + m$, due to space limit, we ignore the algorithm here. The unicast system we present is $\Psi^{\mathsf{LBC}} = (\mathcal{M}^{\mathsf{LBC}}, \mathcal{S}^{\mathsf{LBC}})$, where LBC implies that our algorithm is based on LBC: the service requester s is charged an amount that is half of the weight of LBC. Following Lemma shows that the relation between the weight of LBC and the VCG payment, and the proof is presented in the appendix.

LEMMA 9. *Assume $\mathbf{d}$ is the declared cost vector of the network, then (1) $|\mathbb{LB}(s, t, \mathbf{d})| \geq \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d})$ for any terminal $v_i \in \mathsf{LCP}(s, t, \mathbf{d})$; (2)$|\mathbb{LB}(s, t, \mathbf{d})| \leq 2 \cdot \mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d})$.*

In Algorithm 2, the charge to the service requestor is half of $|\mathbb{LB}(s, t, \mathbf{d})|$ if s is granted the service. From Lemma 9, following theorem shows the performance guarantee of unicast system Ψ^{LBC}.

THEOREM 10. *The Unicast System Ψ^{LBC} is $\frac{1}{2n}$-perfect.*

PROOF. First, we prove that mechanism $\mathcal{M}^{\mathsf{LBC}} = (\mathcal{O}^{\mathsf{LBC}}, \mathcal{P}^{\mathsf{LBC}})$ is strategyproof. IR is straightforward, and we focus on IC. For each terminal v_i, we discuss it by cases.

Case 1*:* If terminal v_i is selected when reports $d_i = c_i$. In this case, it should satisfy that (1) v_i is on $\mathsf{LCP}(s, t, \mathbf{d})$, (2) $\eta \geq \phi$. From the IR property we knows that v_i gets a non-negative utility and its payment $\mathcal{P}_i^{\mathsf{LBC}}(\eta, \mathbf{d}) = \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty}, \mathbf{d})$ which does not depends on its declared cost. Thus, terminal v_i can not increase its utility by falsely reporting its cost.

Case 2*:* If terminal v_i is not selected when reports cost $d_i = c_i$. In this case, if v_i is not on $\mathsf{LCP}(s, t, \mathbf{d})$, then the payment ensures that v_i has no incentive to lie. Thus, we only consider the case when $v_i \in \mathsf{LCP}(s, t, \mathbf{d})$ and $\phi > \eta$, *i.e.*, the terminal v_i is not selected due to the reason that service requestor t has a valuation

Algorithm 2 Least Bridge Cover-based Unicast System $\Psi^{\mathsf{LBC}} = (\mathcal{M}^{\mathsf{LBC}}, \mathcal{S}^{\mathsf{LBC}})$.

Input: A network $G = (V, E)$, a cost vector $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ where d_i is the declared cost of terminal v_i, source s, service requestor t, and t's valuation η

Output: Ψ^{LBC}.

1: Compute the shortest path $\mathsf{LCP}(s,t,\mathbf{d})$ and $\mathbb{LB}(s,t,\mathbf{d})$. Set $\phi = \frac{|\mathbb{LB}(s,t,\mathbf{d})|}{2}$.
2: **if** $\eta \geq \phi$ **then**
3: Each terminal $v_k \in \mathsf{LCP}(s,t,\mathbf{d})$ is selected and receives a payment $\mathcal{P}_k^{\mathsf{VCG}}(\eta^{=\infty}, c)$; all other terminals are not selected and get a payment 0.
4: t is granted the service and charged ϕ.
5: **else**
6: All terminals are not selected and each terminal receives a payment 0.
7: t is not granted the service and is charged 0.

smaller than ϕ. Recall that ϕ does not depend on d_i when $v_i \in \mathsf{LCP}(s,t,\mathbf{d})$. Thus v_i can not affect ϕ while it is on $\mathsf{LCP}(s,t,\mathbf{d})$. Therefore, terminal v_i can not increase its utility by falsely report its cost.

This proves that $\mathcal{M}$ is strategyproof. Recall that the sharing $\xi^{\mathsf{LBC}}(\eta,\mathbf{d}) = \phi$ does not depend on η, thus $\mathcal{S}^{\mathsf{LBC}}$ is strategyproof. Notice that NPT and CS are straightforward, thus we only prove that the Ψ^{LBC} is $\frac{1}{2n}$-budget-balance. To prove that Ψ^{LBC} is $\frac{1}{2n}$-budget-balance, we need to show that

$$\frac{\mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d})}{2n} \leq \xi^{\mathsf{LBC}}(\eta,\mathbf{d}) \leq \mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d}). \tag{2}$$

If $\sigma^{\mathsf{LBC}}(\eta,\mathbf{d}) = 0$, then $\xi^{\mathsf{LBC}}(\eta,\mathbf{d}) = 0$ and $\mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d}) = 0$. The Inequality (2) trivially holds. Thus, we assume that $\sigma^{\mathsf{LBC}}(\eta,\mathbf{d}) = 1$. Consequently, $\mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d}) = \mathbb{P}^{\mathsf{VCG}}(\eta,\mathbf{d})$ and $\xi^{\mathsf{LBC}}(\eta,\mathbf{d}) = \phi$. Recall that $\mathcal{P}_k^{\mathsf{VCG}}(\eta^{=\infty}, c) \leq |\mathbb{LB}(s,t,\mathbf{d})|$ for any $v_k \in \mathsf{LCP}(s,t,\mathbf{d})$. Thus, $\mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d}) = \mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d}) \leq n \cdot |\mathbb{LB}(s,t,\mathbf{d})| = 2n \cdot \phi$. Reorganize it we have $\xi^{\mathsf{LBC}}(\eta,\mathbf{d}) = \phi \geq \frac{\mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d})}{2n}$. On the other hand, from Lemma 9, $\xi^{\mathsf{LBC}}(\eta,\mathbf{d}) = \phi = \frac{|\mathbb{LB}(s,t,\mathbf{d})|}{2} \leq \mathbb{P}^{\mathsf{LBC}}(\eta,\mathbf{d})$. This finishes our proof. $\square$

4.2 $\frac{1}{2}$-NE-perfect Unicast System

Although the unicast system Ψ^{LBC} is $\frac{1}{2n}$-perfect, which is asymptotically optimal, it may not be acceptable in certain circumstances. Thus, again, we try to relax the dominant strategy requirement for the terminals. In this section, we design a unicast system that is $\frac{1}{2}$-NE-perfect. With the help of LCPA mechanism and Least Bridge Cover, we have the following unicast system Ψ^{AU}, where AU stands for LCP Auction based Unicast system. The Unicast System Ψ^{AU} works as follows: it first executes the broadcast phase in LCPA. After obtaining the vector $\mathbf{h}$, it punishes the terminals in $\mathsf{LCP}(s,t,\mathbf{b}) - \mathsf{LCP}(s,t,\mathbf{h})$ for bidding too high for the second round. The punishment for bidding too high can be adjusted by parameters γ for different practical implementation needs. It sends the packet if and only if service requestor t's valuation is greater than *half* cost of the least bridge cover. All the relay terminals on $\mathsf{LCP}(s,t,\mathbf{h})$ receives h_i if t receives the service and 0 otherwise. All relay terminals that are not on $\mathsf{LCP}(s,t,\mathbf{h})$ receives 0 payment. Algorithm 3 describe the unicast System Ψ^{AU} in details.

Regarding the unicast system Ψ^{AU}, we have the following lemma.

LEMMA 11. *There exist some Nash Equilibria for terminals in unicast system $\Psi^{AU} = (\mathcal{M}^{AU}, \mathcal{S}^{AU})$.*

Algorithm 3 Least Cost Path Auction-based Unicast System $\Psi^{\mathsf{AU}} = (\mathcal{M}^{\mathsf{AU}}, \mathcal{S}^{\mathsf{AU}})$.

Input: A network $G = (V, E)$, source s, requestor t, t's declared valuation η, $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$, a parameter γ.

Output: Ψ^{AU}.

1: Execute the broadcast phase in Algorithm 1 (Line 1 to 5). Notice the $\mathbf{h}$ is the vector obtained on Line 5.
2: **for** each terminal $v_i \in \mathsf{LCP}(s,t,\mathbf{b}) - \mathsf{LCP}(s,t,\mathbf{h})$ **do**
3: Set $\mathcal{P}_i^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{LCPA}}(\eta^{=\infty}, \widetilde{\mathbf{b}}) - \gamma \cdot |b_i' - b_i|$
4: Set $\phi = \frac{|\mathbb{LB}(s,t,\mathbf{b})|}{2}$.
5: **if** $\phi \leq \eta$ **then**
6: Set $\sigma^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = 1$, and $\xi^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = \phi$.
7: **for** each terminal $v_i \in \mathsf{LCP}(s,t,\mathbf{h})$ **do**
8: Set $\mathcal{O}_i^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = 1$, $\mathcal{P}_i^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = \mathcal{P}_i^{\mathsf{AU}}(\eta^{=\infty}, \widetilde{\mathbf{b}}) + h_i$.
9: Set $\Psi^{\mathsf{AU}} = (\mathcal{M}^{\mathsf{AU}}, \mathcal{S}^{\mathsf{AU}})$, and output Ψ^{AU}.

PROOF. If $\eta < \mathsf{LCP}(s,t,\mathbf{c})$, then bid $\langle \mathbf{c}, \mathbf{c} \rangle$ is a NE. If $\eta \geq \mathsf{LCP}(s,t,\mathbf{c})$, then similar the proof to Theorem 2, $\langle \mathbf{c}, \mathbf{c} \rangle$ is a also NE. The proof is omitted here due to space limit. $\square$

LEMMA 12. *If $\widetilde{\mathbf{b}}$ is a NE for unicast system Ψ^{AU}, then (1) $\mathbf{b} = \mathbf{c}$; (2) $LCP(s,t,\mathbf{b}) = LCP(s,t,\mathbf{h})$.*

PROOF. The key observation is that $\phi = \frac{|\mathbb{LB}(s,t,\mathbf{b})|}{2}$ does not depend on any terminal v_i on $\mathsf{LCP}(s,t,\mathbf{b})$. The detailed proof is similar to Lemma 3 and is omitted here. $\square$

THEOREM 13. *Ψ^{LCPA} is $1/2$-NE-perfect with ϵ additive for any fixed positive ϵ.*

PROOF. We discuss by cases. If $\eta < \phi$, then $\mathbb{P}^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = \epsilon$ and $\xi^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = 0$. If $\eta \geq \phi$, then $\xi^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}}) = \phi = \frac{|\mathbb{LB}(s,t,\mathbf{b})|}{2}$. From Theorem 4 and Lemma 5, $|\mathbb{LB}(s,t,\mathbf{b})| \geq \nu(\mathbf{c}) = \mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\min}) - \epsilon \geq \frac{\mathbb{P}(\eta^{=\infty}, \widetilde{\mathbf{b}}^{\max})}{2} - \epsilon \geq \frac{\mathbb{P}^{\mathsf{AU}}(\eta, \widetilde{\mathbf{b}})}{2} - \epsilon$. This proves that Ψ^{AU} is $\frac{1}{2}$-budget-balanced with ϵ additive. For a given fix $\widetilde{\mathbf{b}}$, the sharing $\xi^{\mathsf{AU}}(\cdot)$ does not depend on η, thus $\mathcal{S}^{\mathsf{AU}}$ is strategyproof. The NPT and NE-CS property are straightforward, thus Ψ^{AU} is NE-perfect with ϵ additive. $\square$

5. UNICAST SYSTEM IMPLEMENTATION

In Section 4.2, we discuss how to design the unicast system that is NE-perfect. There are quite a few issues should be addressed before the unicast mechanism can be implemented as a protocol in practice.

5.1 Multiple Service Requestors and Broadcast Phase Implementation

In Section 4.2, we only consider the routing between a fixed service requestor and the source. However, there are probably more than one service requestors. The very naive way is to request every terminal to bid a cost b_i for every possible service requestor in the broadcast phase. In the worst case, every terminal need to bid $O(n)$ bids and the source need to collect $O(n^2)$ bids, where n is the number of the terminals. This is not practical and if possible at least not efficient. From Lemma 12, we conclude that $\mathbf{b} = \mathbf{c}$ for any service requestor. Thus, each terminal only needs to send the first bid to the source exactly once as long as its cost does not change.

Another key issue in the broadcast phase is how to broadcast the packet after the the bids are elicited from the terminals. One naive way is that we broadcast the dummy packet for every service request from any service requestor, as done in the Ψ^{AU}. However, notice that the reason for the broadcast is that we need every terminal to send a "dummy" packet of certain size, which can contain any content. Thus, instead of broadcasting the the packet with certain size for each service requestor, we could implement the broadcasting phase in a tricky way: we request each terminal to send out a packet of certain size and pay them certain amount of money periodically regardless of the service request in the network. A possible implementation of the broadcast is presented in Algorithm 4.

Algorithm 4 Broadcasting Phase of $\Psi^{\mathsf{AU}} = (\mathcal{M}^{\mathsf{AU}}, \mathcal{S}^{\mathsf{AU}})$.

1: The source s collects a bid b_i for each terminal v_i.
2: Periodically, say every δ minutes, the source pays each terminal $f_i(s,t,\mathbf{b}) = \tau_i(b_{-i}) \cdot \left[b_u \cdot (n \cdot b_u - \sum_{v_j \in G - v_i} b_j) - \frac{b_i^2}{2}\right]$.
3: Every δ minutes, every terminal sends a "dummy" packet of size $\rho = \tau_i(b_{-i}) \cdot (n \cdot b_u - \sum\limits_{v_i \in G} b_i)$ to all its neighbors. Notice that the terminal sends the packet using a broadcast manner, thus the cost spent for each terminal only depends on the size instead of the number of the neighbors.
4: Each terminal v_i keeps a variable a_{ij} for each of its neighbors v_j to indicate how many times v_j has broadcasted since last reset.
5: Each terminal v_i resets $a_{ij} = 0$ in a certain period of time which is known publicly.

Note that the broadcast phase in Algorithm 4 is totally separated from any specific unicast phase, it has several advantages over the broadcast phase in Algorithm 3: (1) each terminal v_i only needs to bid b_i once as long as there is no cost update. (2) The broadcast phase in Algorithm 4 is not specific to any service requestor, and it can change the function $\tau_i(b_{-i})$ and time parameter δ for different application. (3) The terminals does not need to synchronize exactly in order to broadcast the packet at the same time. The only thing every terminal should guarantee that it broadcasts a packet every δ minutes. (4) The terminal's broadcast activity is monitored by all its neighbors. Thus, it is very easy to ensure the truthful implementation of the broadcast activity at each terminal.

5.2 Unicast Phase Implementation and Fast Convergence

In the unicast phase, the implementation is flexible. Algorithm 5 outlines one possible unicast phase implementation and it is possible that unicast could use some existing routing protocols like DSR [13].

The Algorithm 5 has one drawback: it may take several rounds before certain NE is reached, which depends on whether the terminal are more aggressive or conservative and on the parameter γ. What makes the case worse is that the topology and cost could change overtime. Thus, instead of sending a FAIL or ACK message right away to the service requestor, the sender could send the BID-REQ several times until $\mathsf{LCP}(s,t,\mathbf{h}) = \mathsf{LCP}(s,t,\mathbf{b})$ and $|\mathsf{LCP}(s,t,\mathbf{h})| = |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$ for every terminal v_i on $\mathsf{LCP}(s,t,\mathbf{b})$ *i.e.*, it gives a chance to the terminals on the shortest path to reach some NE. In the meanwhile, the penalty $\gamma \cdot |b_i' - b_i|$ should be deducted from the v_i's payment in every BID-REQ attempt in order to converge fast. Recall that, although there may have multiple BID-REQs for the terminals on the LCP, the packet does not get sent before some NE is reached.

5.3 Other Implementation Issues

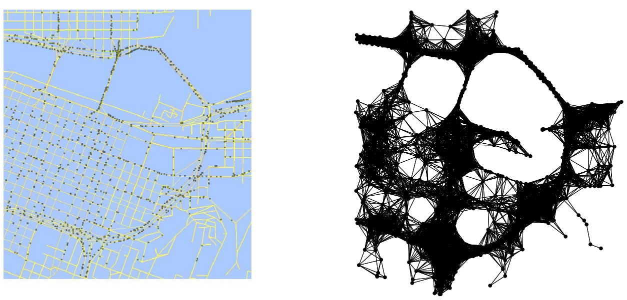
Algorithm 5 Unicast Phase of $\Psi^{\mathsf{AU}} = (\mathcal{M}^{\mathsf{AU}}, \mathcal{S}^{\mathsf{AU}})$.

1: When a service requestor t wants to send or receive some packets from the source, it first sends a routing request packet REQ with its valuation η.
2: Upon receiving the REQ packet, we compute the LCP based on $\mathbf{b}$. It then prepares a BID-REQ packets containing the terminals on the LCP, and sends this BID-REQ back to t along the LCP.
3: Whenever a terminal receives a BID-REQ that contains itself, it prepares a NEW-BID packet that contains its second bid and sends back to s.
4: Whenever the access point s has received all the NEW-BID packets from the terminals on the $\mathsf{LCP}(s,t,\mathbf{b})$, it executes Algorithm 3. It charges terminals accordingly.
5: **if** $\phi \le \eta$ **then**
6: It prepares an ACK packet containing the $\mathsf{LCP}(s,t,\mathbf{h})$ and sends back to the service requestor t along $\mathsf{LCP}(s,t,\mathbf{h})$.
7: It charges the service requestor t ϕ.
8: **else**
9: It prepares a FAIL packet and sends back to the service requestor t.
10: Upon receiving an ACK packet, the node t sends the packet along the path $\mathsf{LCP}(s,t,\mathbf{h})$.

Packet Forwarding: In this paper, we mainly focus on how to select the routing path, how to pay the relay terminals and how to charge the service requestor. It is worth to point out that we have not addressed the issues on how to gather the information in the routing discovery phase and how to forward the packet after the computation of the path and payment. In [25], Zhong *et al.* studied the problem of designing strategyproof routing and forwarding protocols in wireless ad hoc networks, and proposed a game-theoretic and cryptographic techniques integrated approach. This integrated approach should be a critical part of a unicast routing system which can deployed in practice. Thus, for the implementation of our unicast system, we will save our effort and borrow the approach proposed by Zhong *et al.* to deal with the forwarding phase. For more details, refer to [25].

Multiple Access Points: Previous, we only study the scenario in which there only exists single access point. It is possible that there exist multiple access points and a service requestor can communicate with *any* one of these access points. However, no much literatures considered this scenario even in the most basic Axiom Model. Fortunately, the multiple access points problem can be reduced to the single access point problem as follows. Assume there exist multiple access points s_i. First, we add a virtual access node s to the network as the *only* access node. Then, we add terminal u as s's neighbor if and only if u is some access point s_i's neighbor. Finally, we remove all actual access points s_i and its incident edges from the network.

Budget Imbalance Coverage: Note that no matter for perfect system or NE-perfect system, we are not able to achieve budget balance. In other words, the central authority may lose money from time to time due to any service request. The budget imbalance could be covered by the monthly fee or some form of tax. However, the root of cause of the budget imbalance rises from the notation of budget balance: the competitiveness requires that the central authority can *never* retrieve more than the total payment to relay terminals from the service requestor. If the competitiveness does not satisfied, then it is possible that other central authorities could comes compete to lower the price for service requestor. Obviously, the requirement of the competitiveness could be relax reasonable to allow central authority to earn some money in certain cases and lose some money in other cases, while keep the expected budget balanced. As evident from our simulation below, the BBF of our unicast system Ψ^{AU} is alwasy very close to 0.5 while Ψ^{LBC} could varies from 0.1 to 0.5. By simply modifying $\phi = |\mathbb{LB}(s,t,\mathbf{b})|$, the unicast system Ψ^{auni} is budget balanced almost under any circumstance in random wireless network and practice. However, for Ψ^{duni}, even for the same wireless network with terminal mobility, we are not able to modify the system to make the expected budget balanced. This showes that our unicast system Ψ^{AU} not only has a much better theoretical guarantee than Ψ^{LBC}, but also works much better in real world wireless networks.



Positions of wireless devices   Topology of the network

Figure 6: A snapshot $7:10AM$ of the wireless mobile devices and the wireless network .

6. SIMULATION STUDY

In Section 3, we prove that the total payment of LCPA mechanism is not greater than the total payment of the VCG mechanism, and in certain cases the total payment of LCPA is only $O(\frac{1}{n})$ of the total payment of VCG mechanism. However, there is a huge gap between the theoretical bounds. Thus, we are interested in comparing the total payment of LCPA mechanism and VCG mechanism in certain wireless networks. Recall that for the unicast systemΨ^{LBC}, we proved that it is $\frac{1}{2n}$-perfect. We also study the budget balance factor in certain wireless networks.

6.1 Random Wireless Networks

Network Settings: In this simulation, we randomly generate n terminals uniformly in a $2000ft \times 2000ft$ region. The transmission range is computed using the modified CTR topology control protocol [19]: all terminals has the same transmission range, which is set to the critical value for *bi-connectivity*, *i.e.*, to the minimum value r such that the communication graph generated when every terminal with range r is bi-connected. For every terminal v_i, its cost for relaying unit size data is randomly drawn from 1 to 10. The access point is placed in the center of the region in this simulation. Since the additive ϵ could be arbitrary small by tuning the parameter, we disregard it in our simulation.

Comparison of the Total Payment: We first compare the total payment of LCPA mechanism with that of the VCG mechanism. Notice that the total payment of LCPA mechanism varies if the NE are different, and there are could be infinitely many different total payments. In our simulations, we *only* choose the NE $\widetilde{\mathbf{b}}^{\max}$ that maximizes the total payment, which is the worst case. We generate 500 different random networks with 200 terminals and choose 40 different routes per random wireless network. The distribution of total VCG payment and LCPA payment is shown in Figure 5 (a). Notice in Figure 5 (a), the last column gives the number of VCG payment and the maximum LCPA payment that is greater than 10 instead of equaling 10. One can verify that given a fixed value x, the probability that LCPA payment is less than x is always smaller than the probability that VCG payments is less than x.

We also vary the number of terminals from 100 to 500. For each fixed number of terminals, we generate 500 different scenarios and randomly choose 50 different routes in each scenery. The average actual cost, the average weight of the LBC, the average VCG payment and LCPA payment are shown in Figure 5 (b). One striking observation is that the average weight of LBC is almost not distinguishable from the average NE payment, and the average VCG payment is only a little bit larger than the average NE payment.

Budget Balance Factor Study: Here, we use the same network setting and study the budget balance factor of unicast system Ψ^{LBC}. Note that for the unicast system Ψ^{AU} and Ψ^{LBC}, $\phi = \frac{|\mathbb{LB}(s,t,\mathbf{b})|}{2}$ is exactly the same. This is important since for any service requestor t, it always has the same charge in system Ψ^{LBC} and system Ψ^{AU}, which makes the comparison very *fair*. On the other hand, we use the $\widetilde{\mathbf{b}}^{\max}$ as the NE to compute the total payment for Ψ^{AU}, which is the worst case budget balance factor. Figure 5 (c) shows the comparison of the BBFs of the unicast system Ψ^{LBC} and Ψ^{AU}. Notice that the BBF of system Ψ^{AU} is $\frac{1}{2}$ under any case. The average BBF in unicast system Ψ^{LBC} is around 0.45 which is a little smaller than the BBF in unicast system Ψ^{AU}. However, in the worst case, the minimum BBF in unicast system Ψ^{AU} is almost the same as the average BBF while the minimum BBF in unicast system Ψ^{LBC} is smaller than 0.2. Thus, the unicast system Ψ^{AU} can guarantee a much larger BBF in the worst case than unicast system Ψ^{LBC}.

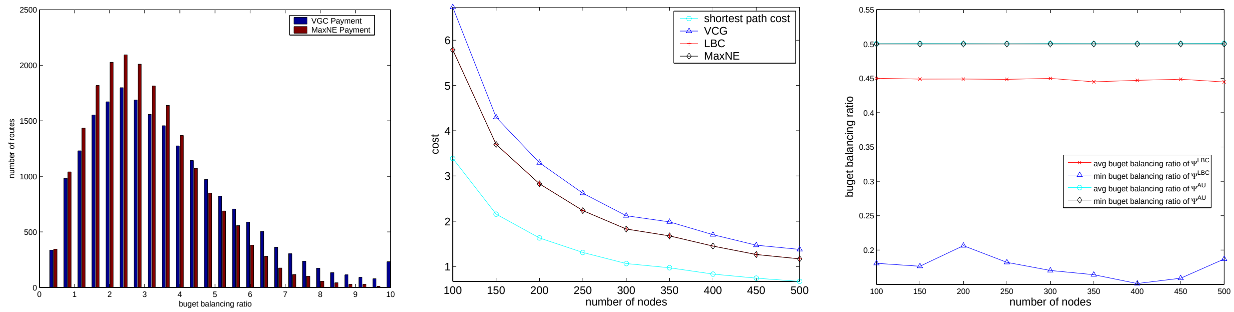
6.2 Wireless Networks in Portland

Network Setup: In this section, we make use of the real snap shots of potential wireless mobile devices in the downtown Portland. These snap shots are based on the movement of a synthetic population created by the Transims software that is statistically indistinguishable from US census data. The three snap shots give the positions of the wireless mobile devices at time $7:00AM$, $7:10AM$ and $7:13AM$ and Figure 6 shows the road map and the position of the mobile devices at $7:10AM$. In this simulation we fix the transmission of the wireless devices to 200, which results in an almost bi-connected wireless communication graph. It is not very difficult to observe that there are several devices that are separated from the others. We will remove these isolated devices and consider the main bi-connected component. In the snapshots, we place the access point at one of five fix positions.

Budget Balance Factor Study: In these snapshots, the cost of the wireless devices is randomly drawn from 1 to 10. We choose every wireless device that is not directly connected to the access point as a potential service requestor an compare the budget balance factors in unicast system Ψ^{LBC} and Ψ^{AU}. Figure 7 shows the distribution of the BBFs in three different snapshots.

It is not difficult to observe that the average BBF of Ψ^{AU} are near 0.5 in all snapshots, while the average BBF of Ψ^{LBC} could vary from 0.33 to 0.39. Furthermore, the minimum BBF of Ψ^{AU} are close to 0.5 for both snapshots while the minimum BBF of Ψ^{LBC} is close to 0.2. Both the random wireless networks and real world snap shot show that the average BBF of Ψ^{LBC} is not very bad, which is between 0.3 to 0.45. However, the minimum BBF of Ψ^{LBC} could be as small as 0.2, while the minimum BBF of Ψ^{AU} is around 0.5 almost for sure.

7. CONCLUSION

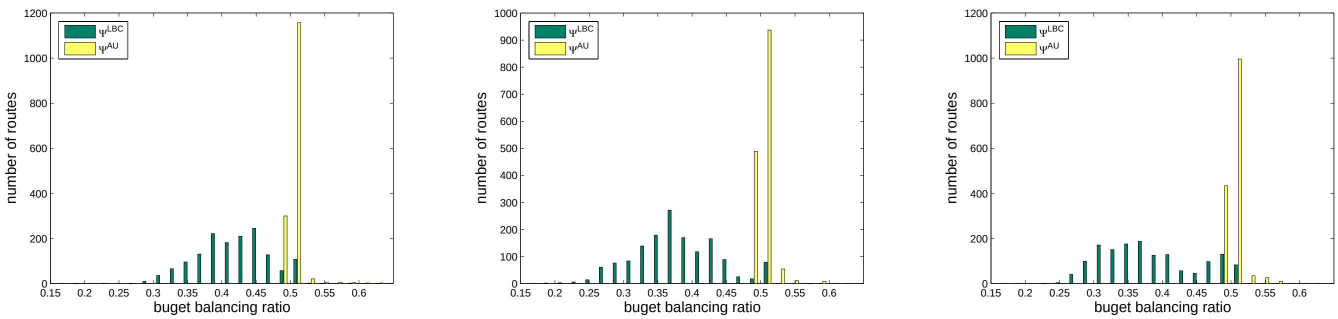


VCG payment and LCPA payment in wireless network with 200 nodes

VCG payment, LCPA payment the actual cost, and weight of LBC

the comparison of the BBFs of the unicast systems Ψ^{LBC} and Ψ^{AU}

Figure 5: Experimental Results in Random Wireless Networks



The BBFs in the $7:00AM$ snapshot.

The BBFs in the $7:10AM$ snapshot.

The BBFs in the $7:13AM$ snapshot.

Figure 7: Budget Balance Factor of Ψ^{LBC} and Ψ^{AU} in three real world snapshot.

In this paper, we study wireless multi-hop networks consisting of selfish, non-cooperative wireless terminals and service requestor. We propose a set of optimal unicast routing systems (OURS) with proved performance guarantee. We first consider the unicast system in the Axiom Model. Since the the budget balance factor of VCG mechanism could be as small as $\frac{1}{n}$ (albeit it is strategyproof and social efficient), we propose the LCPA mechanism that provably reduces the inevitable overpayment by achieving Nash equilibria for the relay terminals. We then consider the unicast system in the Valuation Model in which both the relay terminals and the service requestor could be selfish. For strategyproof and social efficient system in this setting, we prove that no system can guarantee that the access point can retrieve more than $\frac{1}{n}$ of the total payment to the relay terminals; and we present a strategyproof and social efficient unicast system Ψ^{LBC} that collects a fraction $\frac{1}{2n}$ of the total payment which is thus asymptotically optimum. We also propose a social efficient unicast system based on NE solution that achieves a constant budget balance factor.

There are a number of interesting problems left. We mainly studied social efficient mechanisms, and considered the trade-offs between budget balance factor and strategyproofness. It is interesting to design mechanisms that are budget balanced and strategyproof with the best possible social efficiency; or design social efficient and budget balanced mechanisms using Nash Equilibria solution concept.

8. REFERENCES

[1] ANDEREGG, L., AND EIDENBENZ, S. Ad hoc-vcg: a truthful and cost-efficient routing protocol for mobile ad hoc networks with selfish agents. In *9th ACM Mobicom*, 2003.

[2] ARCHER, A., AND TARDOS, E. Frugal path mechanisms. In *13th ACM-SIAM Symposium on Discrete algorithms*, 2002.

[3] BLAZEVIC, L., BUTTYAN, L., CAPKUN, S., GIORDANO, S., HUBAUX, J. P., AND BOUDEC, J. Y. L. Self-organization in mobile ad-hoc networks: the approach of terminodes. *IEEE Communications Magazine 39*, 6 (June 2001).

[4] BUTTYAN, L., AND HUBAUX, J. Stimulating cooperation in self-organizing mobile ad hoc networks. *Mobile Networks and Applications 5*, 8 (Oct. 2003).

[5] BUTTYAN, L., AND HUBAUX, J. P. Enforcing service availability in mobile ad-hoc wans. In *ACM Mobihoc*, 2000.

[6] CLARKE, E. H. Multipart pricing of public goods. *Public Choice* (1971), 17–33.

[7] COUTO, D. S. J. D., AGUAYO, D., BICKET, J., AND MORRIS, R. A high-throughput path metric for multi-hop wireless routing. In *Proc. of 9th ACM Mobicom*, 2003.

[8] EIDENBENZ, S., RESTA, G., AND SANTI, P. Commit: A sender-centric truthful and energy-efficient routing protocol for ad hoc networks with selfish nodes. In *Proc. of 5th IEEE International Workshop on Algorithms for Wireless, Mobile, Ad Hoc & Sensor Networks, IPDPS*, 2005.

[9] GREEN, J., AND LAFFONT, J. J. Characterization of satisfactory mechanisms for the revelation of preferences for public goods. *Econometrica* (1977), 427–438.

[10] GROVES, T. Incentives in teams. *Econometrica* (1973), 617–631.

[11] IMMORLICA, N., KARGER, D., NIKOLOVA, E., AND SAMI, R. First-price path auctions. In *Proceedings of 6th ACM Conf. on Electronic Commerce*, 2005.

[12] JAKOBSSON, M., HUBAUX, J.-P., AND BUTTYAN, L. A micro-payment scheme encouraging collaboration in multi-hop cellular networks. In *Proceedings of Financial Cryptography* (2003).

[13] JOHNSON, D., MALTZ, D., AND BROCH, J. The Dynamic Source Routing Protocol for Multihop Wireless Ad Hoc Networks. In *Ad Hoc Networking*, by C.E. Perkins (Edt), 139–172. Addison-Wesley, 2001.

[14] KAO, M.-Y., LI, X.-Y., AND WANG, W. Towards truthful mechanisms for binary demand games: A general framework. In *Proceedings of 6th ACM Conf. on Electronic Commerce* (2005).

[15] KARLIN, A., KEMPE, D., AND TAMIR, T. Beyond vcg: Frugality of truthful mechanisms. In *Proc. of IEEE FOCS* (2005).

[16] LEHMANN, D., OCALLAGHAN, L. I., AND SHOHAM, Y. Truth revelation in approximately efficient combinatorial auctions. *Journal of ACM 49*, 5 (2002), 577–602.

[17] MU'ALEM, A., AND NISAN, N. Truthful approximation mechanisms for restricted combinatorial auctions: extended abstract. In *Proceedings of 18th National Conference on Artificial Intelligence* 2002.

[18] NISAN, N., AND RONEN, A. Algorithmic mechanism design. In *31st ACM Symp. on Theory of Computing*, 1999.

[19] P.SANTI, AND BLOUGH, D. The critical transmitting range for connectivity in sparse wireless ad hoc networks. *IEEE Transcations on Mobile Computing 2*, 1 (2003), 25–39.

[20] SRINIVASAN, V., NUGGEHALLI, P., CHIASSERINI, C. F., AND RAO, R. R. Cooperation in wireless ad hoc wireless networks. In *Proc. of IEEE Infocom* (2003).

[21] VICKREY, W. Counterspeculation, auctions and competitive sealed tenders. *Journal of Finance* (1961), 8–37.

[22] WANG, W., LI, X.-Y., AND WANG, Y. Truthful multicast in selfish wireless networks. In *10th ACM Mobicom* (2004).

[23] WANG, W., LI, X.-Y., AND CHU, X.-W. Nash Equilibria, Dominant Strateies in Routing. In *Workshop for Internet and Network Economics* (2005).

[24] ZHAO, J., AND GOVINDAN, R. Understanding packet delivery performance in dense wireless sensor networks. In *Proc. of ACM SenSys* (2003).

[25] ZHONG, S., LI, L., LIU, Y. G., AND YANG, Y. R. On designing incentive-compatible routing and forwarding protocols in wireless ad-hoc networks. In *Proc. of ACM Mobicom* (2005).

APPENDIX

Lemma 3 Assume that $\widetilde{\mathbf{b}} = \langle \mathbf{b}, \mathbf{b}' \rangle$ is a Nash Equilibrium for LCPA mechanism $\mathcal{M}^{\mathsf{LCPA}}$, where $\mathbf{h}$ is the cost vector obtained in Algorithm 1.

1. $\mathbf{b} = \mathbf{c}$.

2. $\mathsf{LCP}(s,t,\mathbf{c}) = \mathsf{LCP}(s,t,\mathbf{h})$.

3. For any $v_i \in \mathsf{LCP}(s,t,\mathbf{b}) = \mathsf{LCP}(s,t,\mathbf{c})$, $|\mathsf{LCP}(s,t,\mathbf{h})| = |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$.

PROOF. **Statement 1:** For simplicity for our notation, let $\mathsf{P}_1 = \mathsf{LCP}(s,t,\mathbf{b})$ and $\mathsf{P}_2 = \mathsf{LCP}(s,t,\mathbf{h})$. We first prove that $\mathsf{P}_1 = \mathsf{P}_2$. We prove this by studying edges in different groups.

1. $v_i \notin \mathsf{P}_1 \cup \mathsf{P}_2$. In this case, v_i's overall utility is $g_i(\mathbf{b})$, which is maximized when $b_i = c_i$. Since we directly set $h_i = b_i$ for link $v_i \notin \mathsf{P}_1$, $h_i = b_i = c_i$.
2. $v_i \in \mathsf{P}_1$ and $v_i \notin \mathsf{P}_2$. In this case, v_i's overall utility is $g_i(\mathbf{b}) - \gamma \cdot |b'_i - b_i|$, which is maximized when $b_i = b'_i = c_i$. Thus, $h_i = b'_i = b_i = c_i$.
3. $v_i \in \mathsf{P}_2$ and $v_i \notin \mathsf{P}_1$. In this case, we have $h_i = b_i$.

Then we can conclude that for any node v_j that is not in P_2, $h_j = c_j = b_j$. Thus,

$$\omega(\mathsf{P}_2,\mathbf{h}) - \omega(\mathsf{P}_1,\mathbf{h}) = \sum_{v_j \in \mathsf{P}_2 - \mathsf{P}_1} h_j - \sum_{v_j \in \mathsf{P}_1 - \mathsf{P}_2} h_j$$

$$\geq \sum_{v_j \in \mathsf{P}_2 - \mathsf{P}_1} b_j - \sum_{v_j \in \mathsf{P}_1 - \mathsf{P}_2} c_j = \sum_{v_j \in \mathsf{P}_2 - \mathsf{P}_1} b_j - \sum_{v_j \in \mathsf{P}_1 - \mathsf{P}_2} b_j$$

$$= \sum_{v_i \in \mathsf{P}_2} b_j - \sum_{v_j \in \mathsf{P}_1} b_j = \omega(\mathsf{P}_2,\mathbf{b}) - \omega(\mathsf{P}_1,\mathbf{b}) \geq 0.$$

This implies that $\omega(\mathsf{P}_1,\mathbf{b}) = \omega(\mathsf{P}_2,\mathbf{b})$ and $\omega(\mathsf{P}_1,\mathbf{h}) = \omega(\mathsf{P}_2,\mathbf{h})$. Thus, $\mathsf{P}_1 = \mathsf{P}_2$.

Then we consider all remaining edges, *i.e.*, $v_i \in \mathsf{P}_2 \bigcap \mathsf{P}_1$. Note that $\mathsf{P}_2 \bigcap \mathsf{P}_1 = \mathsf{P}_1$ since $\mathsf{P}_1 = \mathsf{P}_2$. If $b_i \geq c_i$, then by declaring c_i, v_i's utility in the broadcast phase increases. In the meanwhile, v_i is still on P_1, impling that its utility in the unicast phase does not change. Thus, $b_i \leq c_i$ for each terminal on P_1. If v_i is on P_1 and $b_i < c_i$, then it can increase its utility from broadcast phase by bidding $b_i = c_i$. In the meanwhile, $b_i \leq c_i$ for each terminal v_i on P_1 and $b_i = c_i$ for other terminals. Thus, v_i can guarantee that it is still on the LCP when it declared c_i. Therefore, v_i can increase its overall utility by bidding $b_i = c_i$, which contradicts the definition of the Nash Equilibrium. This finishes the proof of first statement.

Statement 2: The second statement is straightforward from the definition and the proof is omitted here.

Statement 3: Assume that terminal $v_i \in \mathsf{LCP}(s,t,\mathbf{b}) = \mathsf{P}_1$, if $|\mathsf{LCP}(s,t,\mathbf{h})| < |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$, then by bidding $\mathbf{b}' + \delta$ for a sufficient small δ such that $|\mathsf{LCP}(s,t,\mathbf{h}'|^i b'_i + \delta)| < |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h}')|$, its utility increases by δ. Thus, $|\mathsf{LCP}(s,t,\mathbf{h})| \geq |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$. On the other hand, from the definition of the LCP, $|\mathsf{LCP}(s,t,\mathbf{h})| \leq |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$. Thus, $|\mathsf{LCP}(s,t,\mathbf{h})| = |\mathsf{LCP}_{-v_i}(s,t,\mathbf{h})|$ for every $v_i \in \mathsf{P}_1$. □

Lemma 9 Assume $\mathbf{d}$ is the declared cost vector of the network, then (1) $|\mathbb{LB}(s,t,\mathbf{d})| \geq \mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$ for any terminal $v_i \in \mathsf{LCP}(s,t,\mathbf{d})$; (2)$|\mathbb{LB}(s,t,\mathbf{d})| \leq 2 \cdot \mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$.

PROOF. **Statement 1:** From the definition of the bridge cover, for each terminal $v_i \in \mathsf{LCP}(s,t,\mathbf{d})$, there must exist a bridge $\mathrm{B} \in \mathbb{LB}(s,t,\mathbf{d})$ such B covers v_i. Since the path composed of $\mathsf{LCP}(s,s(\mathrm{B}),\mathbf{d}), \mathrm{B}$ and $\mathsf{LCP}(t(\mathrm{B}),t,\mathbf{d})$ is a path that does not contain v_i,

$$|\mathsf{LCP}(s,s(\mathrm{B}),\mathbf{d})| + |\mathrm{B}(\mathbf{d})| + |\mathsf{LCP}(t(\mathrm{B}),t,\mathbf{d})| \geq |\mathsf{LCP}_{-v_i}(s,t,\mathbf{d})|.$$

Thus, $\mathcal{P}_i^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d}) = |\mathsf{LCP}_{-v_i}(s,t,\mathbf{d})| - |\mathsf{LCP}(s,t,\mathbf{d})| + d_i \leq |\mathsf{LCP}(s,s(\mathrm{B}))| + |\mathrm{B}(\mathbf{d})| + |\mathsf{LCP}(t(\mathrm{B}),t,\mathbf{d})| - |\mathsf{LCP}(s,t,\mathbf{d})| + d_i = |\mathrm{B}(\mathbf{d})| - |\mathsf{LCP}(s(\mathrm{B}),t(\mathrm{B}),\mathbf{d})| + d_i \leq |\mathrm{B}(\mathbf{d})| \leq |\mathbb{LB}(s,t,\mathbf{d})|$.

Statement 2: In order to prove the second part, we introduce the min k-flow node disjoint paths $\mathcal{F}_k(\mathbf{d})$ for a network when the declared cost vector is $\mathbf{d}$. Note that $\mathcal{F}_1(\mathbf{d}) = \mathsf{LCP}(s,t,\mathbf{d})$. In [11], Immorlica *et al.* proves that $|\mathcal{F}_2(\mathbf{d})| - |\mathcal{F}_1(\mathbf{d})| \leq \mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$. Notice that $\mathcal{F}_2(\mathbf{d}) - \mathcal{F}_1(\mathbf{d})$ is also a bridge cover for $\mathsf{LCP}(s,t,\mathbf{d})$. Thus $|\mathcal{F}_2(\mathbf{d}) - \mathcal{F}_1(\mathbf{d})| \geq |\mathbb{LB}(s,t,\mathbf{d})|$. Combining this with the fact that $|\mathcal{F}_2(\mathbf{d})| \geq 2|\mathcal{F}_1(\mathbf{d})|$, we obtain that $|\mathbb{LB}(s,t,\mathbf{d})| \leq |\mathcal{F}_2(\mathbf{d}) - \mathcal{F}_1(\mathbf{d})| \leq |\mathcal{F}_2(\mathbf{d})| \leq 2 \cdot (|\mathcal{F}_2(\mathbf{d})| - |\mathcal{F}_1(\mathbf{d})|) \leq 2 \cdot \mathbb{P}^{\mathsf{VCG}}(\eta^{=\infty},\mathbf{d})$. This finishes our proof. □