Multicast Capacity Scaling for Cognitive Networks: General Extended Primary Network

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***Abstract*—We study the capacity scaling laws for the *cognitive network* that consists of the *primary hybrid network* (PhN) and *secondary ad hoc network* (SaN). PhN is further comprised of an ad hoc network and a base station based (BS-based) network. SaN and PhN are overlapping in the same deployment region, operate on the same spectrum, but are independent with each other in terms of communication requirements. The primary users (PUs), *i.e.*, the ad hoc nodes in PhN, have the priority to access the spectrum. The secondary users (SUs), *i.e.*, the ad hoc nodes in SaN, are equipped with cognitive radios, and have the functionalities to sense the idle spectrum and obtain the necessary information of primary nodes in PhN. We assume that PhN adopts one out of three classical types of strategies, *i.e.*, *pure ad hoc strategy*, *BS-based strategy*, and *hybrid strategy*. We aim to directly derive multicast capacity for SaN to unify the unicast and broadcast capacity under two basic principles: (1) The throughput for PhN *cannot* be undermined in order sense due to the presence of SaN. (2) The protocol adopted by PhN does *not* alter in the interest of SaN, anyway. Depending on which type of strategy is adopted in PhN, we design the optimal-throughput strategy for SaN. We show that there exists a threshold of the density of SUs according to the density of PUs beyond which it can be proven that: (1) when PhN adopts the pure ad hoc strategy or hybrid strategy, SaN can achieve the multicast capacity of the same order as it is stand-alone; (2) when PhN adopts the BS-based strategy, SaN can asymptotically achieve the multicast capacity of the same order as if PhN were absent, if some conditions of the relations among the number of SUs, PUs, the destinations of each multicast sessions in SaN, and the base stations in PhN hold.**

***Index Terms*—Multicast capacity, Scaling laws, Cognitive networks, Extended networks, Dense networks, Percolation theory**

I. Introduction

Nowadays, wireless networks are regulated by fixed spectrum assignment policy. A large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum ranges with a high variance in time [1], [2]. The limited available spectrum co-exists with the inefficiency in the spectrum usage. To cope with this problem, dynamic spectrum access with cognitive radio has recently been investigated, which is a novel paradigm, called *cognitive network*, that improves the spectrum utilization by allowing secondary users to exploit the existing wireless spectrum opportunistically without having a negative impact on primary users, *i.e.*, licensed users.

In this paper, we focus on scaling laws of multicast capacity for cognitive networks. We construct the *cognitive network* as a superposition of two independent networks, called *primary network* and *secondary network*, that operate at the same time, space and frequency. The secondary users are assumed to be equipped with cognitive radios and have the functionalities to sense the idle spectrum and obtain the necessary information of primary users [1]–[3]. We set the primary network as a *hybrid network*, denoted by PhN, consisting of base stations (BSs) and ad hoc nodes (primary users, PUs) [4], [5]. We assume the secondary network as an ad hoc network, denoted by SaN. To match the reality of spectrum consumption better, we assume that the network model has a *Pyramid structure*. That is, the number of PUs, which are licensed to access to the spectrum at any time, is relatively less than the number of secondary users (SUs), which can opportunistically access to the spectrum.

Our model has three *novel points* relative to most existing works: (1) Since multicast capacity can be regarded as the general result of unicast and broadcast capacity [6], we directly study the multicast capacity for cognitive networks in order to enhance the generality of this study. (2) Since pure ad hoc networks and BS-based networks (static cellular networks) can be regarded as the special case of hybrid networks in terms of the number of BSs [4], [5], [7]–[11], we consider the case that the primary network is a hybrid network, which further increases the generality of our model. (3) We use the Gaussian channel model [12], [13] that can capture better the nature of wireless channel than other classic interference models, such as *protocol model* and *physical model* [14].

We aim to derive the multicast capacity for SaN under two *basic principles*: (1) The throughput for PhN *cannot* be decreased in order sense due to the presence of SaN. (2) The protocol adopted by PhN will *not* alter anyway because of the presence of SaN. These two basic principles are coincident with the abstract of practical techniques of cognitive networks.

We first derive the upper bounds on multicast capacity for a stand-alone network isomorphical to SaN, called *stand-alone* SaN. Obviously, we can use such upper bounds as that for SaN whatever strategy is adopted by PhN, because under Gaussian channel model PhN and SaN always have negative influence (interference) on each other under the noncooperative communication scheme as long as they share the same spectrum

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at the same time. To compute such upper bounds, we exploit the *homogeneity* property and *randomness* property of network topology, [15].

Our main work is to design multicast strategies for SaN under two principles mentioned above, by which the multicast throughput, *i.e.*, the lower bounds on multicast capacity, for SaN can be achieved of the optimal order matching the upper bounds. We design two types of multicast strategies for SaN. In one type of strategy, we devise the hierarchical multicast routing based on the *highway* system consisting of the *first-class highways* (FHs) and *second-class highways* (SHs), and we use a hierarchical TDMA scheme to schedule those highways. In the other type, we build the routing based on the highway system only comprised of *second-class highways* (SHs), in order to avoid the bottleneck on the accessing path into *highways* for some cases [12]. Combining with the two types of strategies, we obtain the achievable multicast throughput as the lower bounds of multicast capacity for SaN.

How to protect the capacity for PhN from decreasing in order sense is the precondition in the designing of any strategy for SaN. Our solution is to set a *protection area* (PA) for every node in PhN. As one important characteristics different from other related work such as [16]–[18], we allow a PA to be *dynamic* according to the state of the corresponding primary node. Benefitting from the dynamics of PAs, secondary users (SUs) can access opportunistically into the spectrum from both time and space domains. While, static protection areas used in [16]–[18] make some SUs be never served. In our solution, an intuitive view is that: At a certain time, the receiver in PhN can possibly receive data at a rate of the same order as in the scenario where PhN monopolizes the spectrum, as long as all transmitters in SaN are out of a large enough PA of this receiver; similarly, a receiver in SaN can also possibly receive data at the same rate (in order sense) as that for the stand-alone SaN, as long as this receiver is out of all PAs of the *active* transmitters in PhN.

Two main technical challenges of the specific designing of multicast strategies for SaN are listed as follows.

How large of the PAs are optimal with respect to the capacities for both SaN and PhN? As discussed above, the larger PAs are better for protecting the throughput for PhN. Meanwhile, too large PAs will result in decreasing the throughput for SaN. In other words, there is a tradeoff between the throughputs for PhN and SaN in terms of the size of PAs. Furthermore, it is easy to understand that the designing of multicast strategies for SaN depends on the specific strategy adopted by PhN. As a hybrid network, PhN could generally adopt three broad categories of multicast strategies, according to [4], [5], [19]. The first one is the classical BS-based strategy under which communications between any users are relayed by some specific BSs. The second one is the *pure ad hoc strategy*, *i.e.*, the multihop scheme without any BS-supported. The third one is the *hybrid strategy*, *i.e.*, the multihop scheme with BS-supported. According to these three strategies adopted by PhN, we define the appropriate PAs for each PU and BS, and call them *A-Type* PA and *B-Type* PA, respectively. Specifically, under pure ad hoc strategy, the B-Type PAs are never *active*; under BS-based strategy, the B-Type PAs are always *active*; and under hybrid strategy, both A-Type PAs and B-Type PAs might be *active* in a certain time.

How to build the highways, including FHs and SHs? Different from the traditional *highways* in [12], [13], the construction of highways in SaN is more complicated because it is involved with the blocking of some active PAs. For FHs, we design a detouring scheme under which every FH detours the PAs, and we can prove that the produced FHs have a large enough density and large enough capacity to support the relay of data in SaN. For SHs, we design a hierarchical TDMA scheduling scheme by which enough SHs can be scheduled in a constant scheduling period, and all SUs have chance to be served via accessing to the SHs, except when PhN adopts BS-based strategy.

As the final result, combining the upper bounds and lower bounds, we show that: (1) When PhN adopts the pure ad hoc strategy or hybrid strategy, the per-session multicast capacity for SaN is of order $\Theta(\frac{1}{\sqrt{mm_d}})$ when $m_d = O(\frac{m}{(\log m)^3})$, and is of order $\Theta(\frac{1}{m})$ when $m_d = \Omega(\frac{m}{\log m})$, where m is the total number of SUs and m_d is the number of destination nodes of each multicast session in SaN. (2) When PhN adopts the BS-based strategy, an infinitesimal fraction of SUs cannot be served. The per-session multicast capacity for SaN is *asymptotically* of the same order as in Case (1).

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we present our main results. We compute the upper bounds of multicast capacity for SaN in Section IV. In Section V, we derive the achievable multicast throughput as the lower bounds on multicast capacity for SaN by designing the specific multicast strategies. In Section VI, we review the related work. In Section VII, we conclude this paper.

II. System Model

A. Network Topology

The network model has a two-layer structure over a square region $\mathcal{A}(n) = [0, \sqrt{n}]^2$. The first layer is the *primary hybrid network* (PhN) consisting of $\Theta(n)$ primary users (PUs, primary ordinary nodes) and $b(n)$ base stations (BSs). In PhN, PUs are placed according to a Poisson point process of unit intensity over the region $\mathcal{A}(n)$; the region $\mathcal{A}(n)$ is partitioned into $b(n)$ square subregions of side length $\sqrt{n/b(n)}$; one base station (BS) is located at the center of each subregion. We assume that BSs are connected via the high-bandwidth wired links that are certainly not the bottleneck throughout the routing. The second layer is the *secondary ad hoc network* (SaN) consisting of $\Theta(m)$ secondary users (SUs, secondary ad hoc nodes). In SaN, SUs are distributed according to a Poisson point process of intensity $\frac{m}{n}$ over the region $\mathcal{A}(n)$. We randomly choose n_s (or m_s) nodes from all PUs (or SUs) as the sources of the multicast sessions in PhN (or SaN), and for each PU v^p (or SU v^s), pick uniformly at random n_d PUs (or m_d SUs) as the destinations. From Chebychev's inequality,

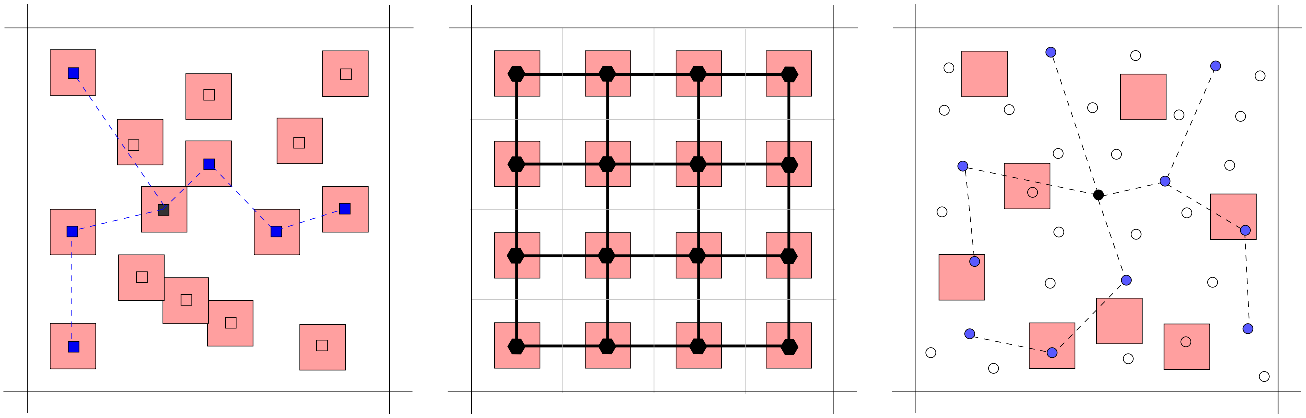


Fig. 1. PhN consists of *PU layer* and *BS layer*; SaN has only one layer, *i.e.*, *SU layer*. (a) The black small square is the source of a given multicast session. The bigger shaded squares are the *A-Type PAs*. (b) The small black hexagons are the BSs that are placed in the center positions of the subregions of area $n/b(n)$. The shaded squares around BSs are the *B-Type PAs*. (c) Dashed lines denote the *Euclidean spanning tree* of a given multicast session.

we can assume that the numbers of PUs and SUs are n and m, respectively, as in [12], [20], which does not change our results in order sense. The following are our basic assumptions.

Assumption 1: PhN operates as if SaN were absent. That is, PhN does not alter its protocol due to SaN anyway.

Assumption 2: Secondary nodes can sense the locations of primary nodes and know the protocols adopted by PhN.

Assumption 3: PhN and SaN are overlapped into a layered network with a *Pyramid structure*. Specifically, $n = o(\frac{m}{\log m})$.

According to Assumption 3, SaN is indeed *dense scaling* [6], [14], [15], [21], [22], while PhN is an extended network [4], [5], [12], [23]–[25]. More discussions about two types of scaling networks can be found in Section II-B of our technical report [26].

B. Communication Model

We assume that all primary users (PUs) and BSs transmit with the constant wireless transmission power P, [4], [5]. Let $\mathcal{V}(\tau)$ denote the set of transmitters in time slot τ. Then, during any time slot τ, a node $v_i \in \mathcal{V}(\tau)$ can communicate with another node v_j via a direct link, over a channel with bandwidth B, of rate

$$R(v_i, v_j; \tau) = B \log\left(1 + \frac{P\cdot\ell(v_i,v_j)}{N_0 + \sum_{v_k\in\mathcal{V}(\tau)/v_i} P\cdot\ell(v_k,v_j)}\right),$$

where the constant $N_0 > 0$ is the ambient noise. The wireless propagation channel typically includes path loss with distance, shadowing and fading effects. As in most related works, such as [12], [13], we assume that the channel gain depends only on the distance between a transmitter and receiver, and ignore shadowing and fading. Following the setting in [12] (defined in Section 3 of [12]), we give PhN and SaN different channel power gain according to their different scaling characteristics. Since PhN is *extended scaling*, the channel power gain $\ell(v_i, v_j)$ is given by $\ell(v_i, v_j) = \min\{1, d_{ij}^{-\alpha}\}$; Since SaN is *dense scaling*, the channel power gain $\ell(v_i, v_j)$ is given by $\ell(v_i, v_j) = d_{ij}^{-\alpha}$. Here, $d_{ij} = d(v_i, v_j) = \|v_i v_j\|$ is the Euclidean distance between two nodes v_i and v_j, $\alpha > 2$ denotes the power attenuation exponent [12].

C. Capacity Definition

Let $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ denote the set of all nodes in the network and let the subset $\mathcal{S} \subseteq \mathcal{V}$ denote the set of source nodes of multicast. Let the number of multicast sessions be $|\mathcal{S}| = n_s$. For each source $v_{\mathcal{S},i} \in \mathcal{S}$, we uniformly choose n_d nodes at random from other nodes to construct $\mathcal{D}_{\mathcal{S},i} = \{v_{\mathcal{S},i_1}, v_{\mathcal{S},i_2}, \cdots, v_{\mathcal{S},i_{n_d}}\}$ as the set of destinations, where obviously $n_d \le n-1$. We call $\mathcal{U}_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \cup \mathcal{D}_{\mathcal{S},i}$ the *spanning set* of multicast session $\mathcal{M}_{\mathcal{S},i}$. Denote $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ as a *rate vector* of the multicast data rate of all multicast sessions. We say a rate vector *asymptotically feasible* if it is $(1,1)$-*feasible* [18].

Based on a *multicast rate vector*, we define the per-session multicast throughput (PMT) as

$$\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = \tfrac{1}{n_s} \cdot \textstyle\sum_{v_{\mathcal{S},i}\in\tilde{\mathcal{S}}(1,1)} \lambda_{\mathcal{S},i}.$$

Furthermore, the PMT $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = \frac{1}{n_s}\cdot\sum_{v_{\mathcal{S},i}\in\tilde{\mathcal{S}}(1,1)} \lambda_{\mathcal{S},i}$ is *asymptotically achievable* if $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \cdots, \lambda_{\mathcal{S},n_s})$ is *asymptotically feasible*.

Definition 1 (Asymptotic Multicast Capacity): The *asymptotic per-session multicast capacity* (Asymp-PMC) of a class of random networks is of order $\Theta(g(n))$ if there are deterministic constants $0 < c < d < +\infty$ such that

$$\lim_{n\to+\infty} \Pr(\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = c\cdot g(n) \text{ is } \textit{Asymp-Achievable}) = 1,$$

$$\liminf_{n\to+\infty} \Pr(\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = d\cdot g(n) \text{ is } \textit{Asymp-Achievable}) < 1.$$

The traditional definition of *per-session multicast capacity* (PMC) [13] can be regarded as a special case of *asymptotic per-session multicast capacity* when $\tilde{\mathcal{S}}(1,1) = \mathcal{S}$.

III. Main Results

We present the upper bounds and lower bounds on multicast capacity for the secondary ad hoc network (SaN), respectively. Finally, combining the lower bounds and upper bounds, we obtain the multicast capacity. We assume that $m_s = \Theta(m)$, where m is the total number of secondary nodes and m_s is the number of multicast sessions in SaN. Such assumption is made in most works related to capacity scaling laws. To simplify the description, let the expression $\phi(n) : [\phi_0(n), \phi_1(n)]$ represent that $\phi(n) = \Omega(\phi_0(n))$ and $\phi(n) = O(\phi_1(n))$.

A. Upper Bounds on Multicast Capacity

We first derive the upper bounds on multicast capacity for SaN as if the primary hybrid network (PhN) were absent. Straightforwardly, such results are also the upper bounds for SaN when PhN works.

Theorem 1: The PMC for SaN is at most of order

$$\bar{\Lambda}^{\mathrm{P}} = \begin{cases} O(\frac{1}{\sqrt{m_d m}}) & \text{when } m_d : [1, \frac{m}{(\log m)^2}] \\ O(\frac{1}{m_d \cdot \log m}) & \text{when } m_d : [\frac{m}{(\log m)^2}, \frac{m}{\log m}] \\ O(\frac{1}{m}) & \text{when } m_d : [\frac{m}{\log m}, m] \end{cases}$$

Here, m_d denotes the number of destination nodes of each multicast session in SaN.

Clearly, the result in Theorem 1 always holds whatever strategy is adopted in PhN.

B. Lower Bounds on Multicast Capacity

We derive the lower bounds on multicast capacity by designing the strategies for SaN corresponding to three classical types of strategies adopted in PhN [4], [5].

1) When PhN Adopts Pure Ad Hoc Strategy: In this case, all secondary users (SUs) involved in the multicast sessions can be served. We have

Theorem 2: The achievable PMT for SaN is of order

$$\Lambda^{\mathrm{P}} = \begin{cases} \Omega(\frac{1}{\sqrt{m_d m}}) & \text{when } m_d : [1, \frac{m}{(\log m)^3}] \\ \Omega(\frac{1}{m_d (\log m)^{\frac{3}{2}}}) & \text{when } m_d : [\frac{m}{(\log m)^3}, \frac{m}{(\log m)^2}] \\ \Omega(\frac{1}{\sqrt{m m_d \log m}}) & \text{when } m_d : [\frac{m}{(\log m)^2}, \frac{m}{\log m}] \\ \Omega(\frac{1}{m}) & \text{when } m_d : [\frac{m}{\log m}, m] \end{cases} \tag{1}$$

2) When PhN Adopts BS-based Strategy: In this case, some SUs are covered by the *B-Type* PAs that are always *active*, then they cannot be served. Under our strategy for SaN, we can ensure that there are at least $\rho_s(m) \cdot m$ multicast sessions of SaN with at least $\rho_d(m) \cdot m_d$ destinations can be served, where $\rho_s(m) \to 1,\ \rho_d(m) \to 1$, as $m \to \infty$.

Theorem 3: The Asymp-achievable PMT for SaN is of order Λ^{P}, where Λ^{P} is defined in (1).

3) When PhN Adopts Hybrid Strategy: In this case, we set SaN to be idle when the down-links and up-links involved with the BSs are scheduled in PhN, and we schedule SaN in the other phases. Under such strategy, we get the achievable multicast throughput of the same order as in Theorem 2.

C. Multicast Capacity for SaN

Combining the upper bounds in Theorem 1 and the lower bounds in Section III-B, we can obtain

Theorem 4: When PhN adopts the pure ad hoc strategy or the hybrid strategy, the PMC for SaN is of order

$$C^{\mathrm{P}} = \begin{cases} \Theta(\frac{1}{\sqrt{m_d m}}) & \text{when } m_d : [1, \frac{m}{(\log m)^3}] \\ \Theta(\frac{1}{m}) & \text{when } m_d : [\frac{m}{\log m}, m] \end{cases} \tag{2}$$

When PhN adopts the BS-based strategy, Asymp-PMC for SaN are also of order C^{P}.

Note that there exists a gap between the upper bounds and lower bounds for the case when $m_d : [m/(\log m)^3, m/\log m]$. We leave out closing this gap for our future work.

IV. Upper Bounds on Multicast Capacity for SaN

In this section, we propose the upper bounds on multicast capacity for SaN when PhN is absent. In Section V, we will prove that such bounds can be achieved indeed.

Therefore, we focus on the stand-alone SaN, where the nodes are distributed into the square region $[0, \sqrt{n}]^2$ according to a Poisson distribution of the density of $\frac{m}{n}$ with $n = o(\frac{m}{\log m})$. Now, we begin to prove Theorem 1. Firstly, we have

Lemma 1: The PMC for SaN is at most of order

$$O(\max\{1/\sqrt{m \cdot m_d}, \log m/m\}).$$

Proof: Please refer to Appendix B-A of [26]. ■

Lemma 2: The PMC for SaN is at most of order

$$\begin{cases} O(\frac{1}{m_d \log m}) & \text{when } m_d = O(\frac{m}{\log m}) \\ O(\frac{1}{m}) & \text{when } m_d = \Omega(\frac{m}{\log m}) \end{cases}$$

Proof: Please refer to Appendix B-B of [26]. ■

Combining Lemma 1 with Lemma 2, we get Theorem 1.

V. Lower Bounds on Multicast Capacity for SaN

Generally, the lower bounds on the capacity can be obtained by designing the specific multicast strategies. To be convenient to describe the strategies, we first recall a notion.

Definition 2 (Scheme Lattice, [27]): Divide a square deployment region of side length $\mathfrak{a}$ into a lattice consisting of square cells of side length $\mathfrak{l}$, we call the lattice *scheme lattice* and denote it as $\mathbb{L}(\mathfrak{a}, \mathfrak{l}, \theta)$, where $\theta \in [0, \frac{\pi}{4}]$ is the minimum angle between the sides of deployment region and those of produced cells.

A. Overview of Multicast Strategy

Denote the strategy for PhN as $\mho$ consisting of routing scheme $\mho^r$ and transmission scheduling $\mho^t$. Assume that every subphase of $\mho^t$, *i.e.*, $\mho^{t_j}$ (for $j = 1, 2, \cdots, \varsigma$) operates under an independent TDMA scheme. Denote the scheduling periods of those TDMA scheme as K_j^2, where $K_j \geq 3$.

We will design the multicast strategy for SaN according to the specific strategy adopted by PhN. We first construct the specific *protection areas* (PAs) for each primary user (PU) and each BS in PhN, and call them *A-Type* PA and *B-Type PA*, respectively. Please see the illustration in Fig. 1(a) and Fig. 1(b). Then, at slot $\tau(\tau_j, j)$, $1 \leq j \leq \varsigma$ and $1 \leq \tau_j \leq K_j^2$, which represents the τ_jth time slot in a scheduling period in phase j, we set the status of PAs of the nodes scheduled in $\tau(\tau_j, j)$ as *active*. Thus, in any time slot τ, the region $\mathcal{A}(n) = [0, \sqrt{n}]^2$ is partitioned into two regions: the *occupied region* $\mathcal{O}(\tau)$, which is the region covered by all *active* PAs in time slot τ, and the *vacant region* $\mathcal{V}(\tau)$, which is the complement of region $\mathcal{O}(\tau)$. Accordingly, we denote the set of the SUs covered by the *occupied region* $\mathcal{O}(\tau)$ or surrounded by the *active* PAs in time slot τ, as the set $\mathcal{P}(\tau)$. Finally, for a given multicast session $\mathcal{M}_{\mathcal{S},i}$ with the source node $v_{\mathcal{S},i}$, when $v_{\mathcal{S},i} \in \bigcap_{\tau(\tau_j,j)} \mathcal{P}(\tau)$, the multicast session $\mathcal{M}_{\mathcal{S},i}$ will be ignored; otherwise, by using the algorithm in [13], we construct the Euclidean spanning tree (EST), denoted by $\mathrm{EST}(\tilde{\mathcal{U}}_{\mathcal{S},i})$, based on the set $\tilde{\mathcal{U}}_{\mathcal{S},i}$, where $\mathcal{U}_{\mathcal{S},i}$ is the *spanning set* and $\tilde{\mathcal{U}}_{\mathcal{S},i} = \mathcal{U}_{\mathcal{S},i} - \bigcap_{\tau(\tau_j,j)} \mathcal{P}(\tau)$. Then, similar to the multicast routing designed in [13], our routing for SaN is guided by the spanning tree $\mathrm{EST}(\tilde{\mathcal{U}}_{\mathcal{S},i})$. The communication of each link in $\mathrm{EST}(\tilde{\mathcal{U}}_{\mathcal{S},i})$ is routed via the highway system similar to that in [24], if applicable. However, intuitively, the routing paths might be broken by the *active* PAs in some (or even all) time slots. Thus, how to deal with such intractability? Is it possible that the optimal throughput for SaN can be achieved? Here, the called optimal order of throughput is the upper bounds of multicast capacity for the stand-alone SaN. Then, given a specific protocol in PhN, we have *three questions* in designing multicast strategies for SaN.

Question 1: How to construct and schedule the *first-class highways* (FHs) and *second-class highways* (SHs) such that, in any time slot when SaN is scheduled, no link along the highways is across the *active* PAs?

Question 2: How large is the density of the highway system in SaN, including the FHs and SHs, if exists?

Question 3: How to ensure our multicast strategy to serve the SUs (or multicast sessions) as much as possible?

Obviously, the status of PAs and the method of constructing the highway system in SaN are determined by the strategy adopted by PhN. Thus, all of three questions should be answered depending on the protocol of PhN. According to the existing works [4], [5], when the TDMA transmission scheduling scheme is adopted in PhN, the strategy for hybrid network can be classified into three types, *i.e.*, *pure ad hoc strategy*, *BS-based strategy* and *hybrid strategy*. Next, we introduce concisely these strategies, and answer the three questions above according to the specific protocol of PhN.

B. When PhN Adopts Pure Ad Hoc Strategy

In PhN, under the pure ad hoc strategy, since no base station is used, all *B-Type* PAs are always *inactive*. For an *A-Type* PA, its status (*active* or *inactive*) is determined by the routing and transmission scheduling adopted by PhN. To achieve the optimal order of throughput for PhN, we assume that the multicast strategy in [24] is adopted in PhN. The strategy is divided into two phases.

Denote the routing scheme in the first phase in PhN as $\mho^{r_1}$ and the transmission scheduling in PhN as $\mho^{t_1}$. In this phase, the strategy is designed based on the scheme lattice $\mathbb{L}(\sqrt{n}, c, \frac{\pi}{4})$, where $c > 0$ is a constant defined in [24]. The routing is constructed based on the *first-class highways* (FHs) consisting of the short links of constant length; and those short links are scheduled by a TDMA scheme. Assume that the constant scheduling period is K_1^2 ($K_1 = 3$ in [24]).

Denote the routing scheme in the second phase in PhN as $\mho^{r_2}$ and the transmission scheduling as $\mho^{t_2}$. In this phase, the strategy is designed based on the scheme lattice $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$, where σ is a constant defined in Lemma 4 of [24] and ϵ_n is an adjusting constant to ensure the value of $\sqrt{n}/(\sigma\sqrt{\log n} - \epsilon_n)$ to be an integer; the routing is constructed based on the *second-class highways* (SHs) consisting of the links of length of order $\Theta(\sqrt{\log n})$; and those links are also scheduled by a TDMA scheme of constant period K_2^2 ($K_2 = 4$ in [24]). An important method is called the *parallel transmission scheduling* under which $\Theta(\log n)$ links initiating from each active cell are simultaneously scheduled.

As in PhN, the highway system in SaN also consists of two levels highways: *first-class highways* (FHs) and *second-class highways* (SHs). We first introduce them from the situation where PhN is not considered, and then extend them to the real situation in which the priority of PhN is inviolable.

1) Highways for SaN absent of PhN: When the PhN is ignored, the highway system can be constructed by the similar method in [24]. The first-class highways (FHs) are indeed the highways constructed in [12]. The second-class highways (SHs) are built without using percolation theory [12].

Existence and Density of FHs: The FHs are constructed and scheduled based on the scheme lattice $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ as illustrated in Fig. 2. Since the distribution of SUs follows a Poisson with mean of c^2 (derived by the intensity $\frac{m}{n}$ times the area of the cell $c^2 \cdot \frac{n}{m}$), the cell in $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ has the same *open* probability as the cell of the lattice in Fig.2 of [12], *i.e.*, $p \equiv 1 - e^{-c^2}$. Let $h = \frac{\sqrt{m}}{\sqrt{2}\cdot c}$. According to Theorem 5 of [12], by choosing a large enough c, there are $\Omega(h)$ paths crossing the network area from left to right, *w.h.p.*, and these can be grouped into disjoint sets of $\delta \cdot \log h$ paths, with each group crossing a rectangle slab of size $\sqrt{n} \times (\kappa \cdot \log h - \epsilon_h) \cdot \sqrt{2 \cdot \frac{n}{m}} \cdot c$, for all $\kappa > 0$, δ small enough, and a vanishingly small ϵ_h so that the side length of each rectangle is an integer.

Therefore, we can divide such rectangle into $\delta \log h$ slices of size $\sqrt{n} \times \varpi(n,m)$, where $\varpi(n,m) = \Theta(\sqrt{\frac{n}{m}})$. Denote the jth slice in the ith slab as $s_h(i,j)$, where $1 \le i \le \frac{h}{\kappa \cdot \log h - \epsilon_h}$ and $1 \le j \le \delta \cdot \log h$. Based on this, we allocate the relay burden of nodes in $s_h(i,j)$ to a specific first-class highway (FH), denoted by $\hbar_h(i,j)$ representing the jth horizontal FH in the ith slab. Similarly, for the vertical case, we can explain the corresponding $s_v(i,j)$ and $\hbar_v(i,j)$, and define the mapping between them.

Existence and Density of SHs: The SHs are constructed and scheduled based on the scheme lattice $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$. For the *dense scaling* network model, the parallel transmission scheduling does not work [24]. Then, in SaN, having no parallel SHs like in PhN [5], there exists only one second-class highway (SH) in each column (or row). Denote each column as $s_v'(i)$, where $1 \le i \le \frac{\sqrt{n}}{\sigma \cdot \sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m}$. Based on this, we allocate the relay burden of nodes in $s_v'(i)$ to a specific second-class highway (SH), denoted by $\hbar_v'(i)$ representing the

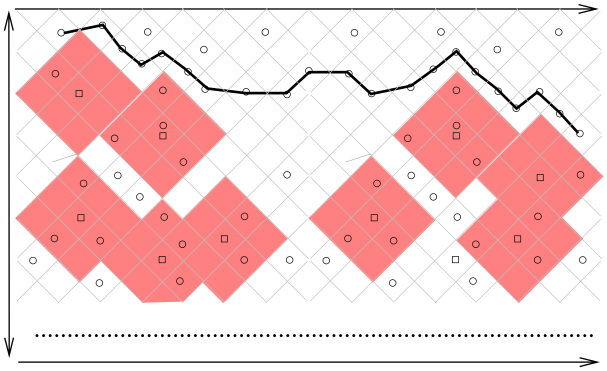


Fig. 2. The cells are of side length $\bar{c} = c \cdot \sqrt{\frac{n}{m}}$. The slab is of side length $l = (\kappa \cdot \log h - \epsilon_h) \cdot \sqrt{2}\bar{c}$. The shaded regions are the *A-Type* PAs. The small square nodes at the center of *A-Type* PAs represent the primary users, and the small circle nodes represent the secondary users. Those cells that contain at least one secondary users and are not shaded are called *non-protected open*.

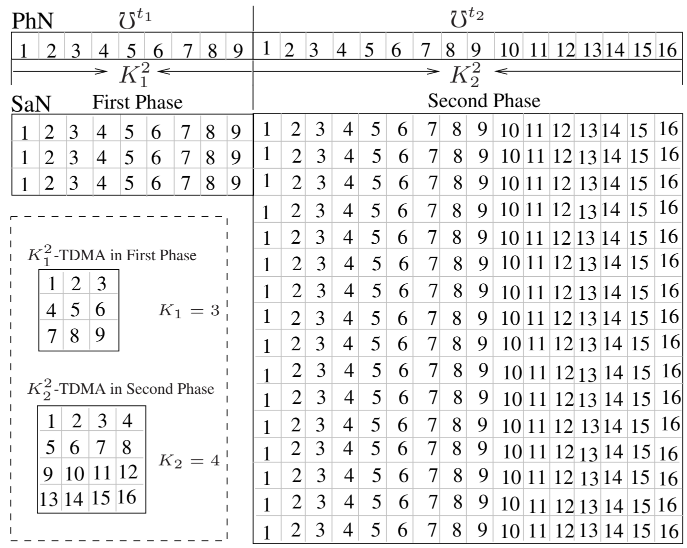


Fig. 3. Illustration of Scheduling Scheme. We describe the case that $K_1 = 3$ and $K_2 = 4$. The scheduling for SaN is divided into two phases corresponding to two phases in PhN. In the first (or second) phase, SaN schedules in sequence $K_1 \times K_1$ (or $K_2 \times K_2$) cells in the scheme lattice $\mathbb{L}(\sqrt{n}, c\sqrt{n/m}, \frac{\pi}{4})$ (or $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot n/m} - \epsilon_m, 0)$) during one period of $3K_1^2$ (or $K_2^4 = K_2^2 \times K_2^2$) slots; that is, each cell will be scheduled continuously 3 (or K_2^2) slots. Remark that, during the continuous K_2^2 slot for each cell in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot n/m} - \epsilon_m, 0)$, the cell is really scheduled only when it is not covered by the *active* PAs.

SH contained in the ith column. Similarly, for the vertical case, we can explain the corresponding $s'_h(i)$ and $\hbar'_h(i)$, and define the mapping between them. Remark that we can use a TDMA with the constant period K_2^2, to schedule the SHs. We will provide the detailed analysis in Section V-B4.

2) Highways for SaN Present of PhN: Consequently, we construct the highway system for SaN based on percolation theory [12], ensuring that no highways in SaN crosses the *active* PAs in any time slot.

Existence and Construction of FHs: The FHs in SaN will be scheduled in the first phase in PhN, *i.e.*, $\mho^{r_1}$ or $\mho^{t_1}$. In this phase, surrounding each PU, we build its *A-Type* PA as a cluster of nine cells of side length $c \cdot \sqrt{\frac{n}{m}}$, as illustrated in Fig 2. In any time slot τ_1, an *A-Type* PA is *active* or *inactive* depends on whether the central primary user (PU) is scheduled (including both transmitting and receiving) or not. Recall that the transmission scheduling of FHs in PhN is a TDMA scheme with constant period K_1^2. Then, in any time slot of the scheduling for FHs in PhN, the number of scheduled cells will be $\frac{1}{K_1^2}$ of the number of all cells, *i.e.*, $\frac{n}{c^2}$. That is, in some time slots, the number of scheduled PUs is of order $\Theta(n)$. Hence, it has no impact on our results in terms of order when the dynamic of the status of *A-Type* PAs in the first phase is ignored, *i.e.*, all *A-Type* PAs are always regarded as *active* in the first phase.

Next, we build the FHs in SaN that co-exists with PhN. We first modify the definition of *open* cells [12]. A cell in $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ is called *non-protected open* if it is nonempty, *i.e.*, it contains at least one SU, and does not belong to any *A-Type* PAs. Please see the illustration in Fig. 2. Then, we have the following lemma.

Lemma 3: When $n = o(m)$, a cell in $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ is *non-protected open* with probability $p_s \to p$ as $n \to \infty$.

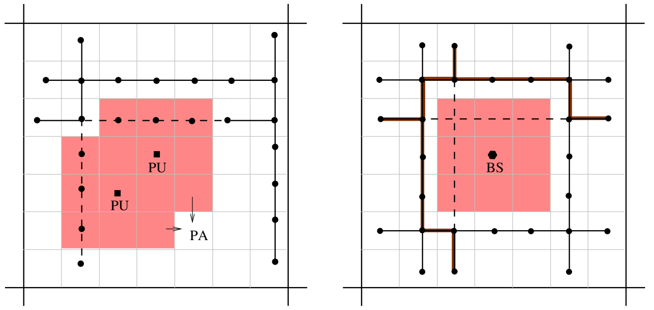
Proof: According to the definition of *non-protected open*, we have $p_s = (1 - e^{-c^2}) \cdot e^{-\frac{9c^2 n}{m}}$, where $e^{-\frac{9c^2 n}{m}}$ is the probability that a cell in $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ is not covered by any PAs. Combining with the condition that $m = \omega(n)$, we obtain that $p_s \to p$ as $n \to \infty$, which completes the proof. ■

By Lemma 3, we can prove the existence of FH in SaN, and obtain the same density of FHs in SaN as that in PhN. Thus, we can use the same notations of FHs in the situation absent of PhN, which will be used in Algorithm 1.

Scheduling of FHs in SaN: Let the transmitting power of SUs in the first phase be $P' \cdot (c\sqrt{\frac{n}{m}})^\alpha$, where $P' \in (0, P_0]$ is a constant. Recall that the constant P_0, defined in Section II-B, is the maximum transmitting power in SaN. Obviously, $P' \cdot (c\sqrt{\frac{n}{m}})^\alpha \in (0, P_0]$. Because the FHs in SaN detour all PAs, the capacity of FHs in PhN can be protected from increasing in terms of order, which is proved in Theorem 5. As illustrated in Fig.3, in the first phase in SaN, the *scheduling unit* is also the cluster of $K_1 \times K_1$ cells. Unlike in PhN, each cell in a *scheduling unit* is scheduled continually 3 slots. By this method, it holds that there is at least one slot out of these three slots during which the nearest distance between the transmitter in PhN and the receiver in SaN is of a constant order.

Scheduling of SHs in SaN: As the new scheme lattice, denoted by $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$, is used, we define the new *A-Type* PA that is a cluster of nine cells in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$ centered at a primary user (PU). Please see Fig. 4(a). Let the transmitting power of SUs in the second phase be $P' \cdot (\sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m)^\alpha$, where $P' \in (0, P_0]$ is a constant. Obviously, it holds that $P' \cdot (\sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m)^\alpha \in (0, P_0]$.

Do the SHs, constructed for SaN absent of PhN, still work now? The following Lemma 4 will answer this question.



(a) SHs Meets *A-Type* PA (b) SHs Meets *B-Type* PA

Fig. 4. SHs built based on $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$. (a) When PhN adopts pure ad hoc strategy, the SHs in SaN need not detour the PAs, but wait for their *inactive* status. (b) When PhN adopts BS-based strategy, since the PAs are always *active*, SHs in SaN have to detour all *B-Type* PAs along the SHs adjacent to the PAs. The bold polylines denote the detouring paths.

Lemma 4: $\bigcap_{\tau_2=1}^{K_2^2} \mathcal{P}(\tau(\tau_2, 2)) = \emptyset$, where $\mathcal{P}(\tau(\tau_2, 2))$ represents the set of SUs covered or surrounded by the *active* PAs in the time slot $\tau(\tau_2, 2)$, *i.e.*, the τ_2th scheduling slot of the second phase in PhN.

Proof: We will use the apagoge to prove this result. Assume that there were an SU, say v, such that the SU $v \in \bigcap_{\tau_2=1}^{K_2^2} \mathcal{P}(\tau(\tau_2, 2))$, in other words, $\bigcap_{\tau_2=1}^{K_2^2} \mathcal{P}(\tau(\tau_2, 2)) \neq \emptyset$. This means that during the second phase in PhN, the distance between any pairs of PUs scheduled in different time slots is of order $O(\sqrt{2} \cdot (\sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m))$, *i.e.*, the length of the diagonal of the *A-Type* PA. On the other hand, since the scheme lattice $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log n} - \epsilon_n, 0)$ is adopted by PhN in the second phase, there must be two PUs that are scheduled in different time slots between which the distance is of order $\Omega(\sqrt{2} \cdot K_2 \cdot (\sigma\sqrt{\log n} - \epsilon_n))$. Furthermore, since $m = \omega(n)$, it holds that $\frac{n}{\log n} = o(\frac{m}{\log m})$, then

$$\sqrt{2} \cdot (\sigma\sqrt{\log m \cdot n/m} - \epsilon_m) = o(\sqrt{2} \cdot K_2 \cdot (\sigma\sqrt{\log n} - \epsilon_n)).$$

There is a contradiction about the distance between specific pairs of PUs. Hence, we get that $\bigcap_{\tau_2=1}^{K_2^2} \mathcal{P}(\tau(\tau_2, 2)) = \emptyset$. ■

This lemma means that for any SU, there is at least one slot out of the scheduling period of the second phase in PhN, *i.e.*, K_2^2 time slots, in which the SU can be possibly scheduled. Recall that K_2^2 is the constant period of TDMA scheme used for SHs in SaN absent of PhN. Hence, we can use a TDMA scheme with the period of $K_2^4 = K_2^2 \times K_2^2$ to schedule the SHs at least once. Since $K_2^2 \in (0, +\infty)$, we can obtain the same capacity, in order sense, of the SHs in SaN regardless of the presence of PhN.

3) Multicast Strategy for SaN: For a given multicast session $\mathcal{M}_{\mathcal{S},i}$ with the source node $v_{\mathcal{S},i}$ and the *spanning set* $\mathcal{U}_{\mathcal{S},i}$, we first construct the Euclidean spanning tree $\mathrm{EST}(\mathcal{U}_{\mathcal{S},i})$ by the method in [13]. Then, we can build the multicast routing tree based on the highway system and spanning tree $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$. More specifically, for each communication pair in $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$, *i.e.*, an edge, the packets will access to the specific FH via the specific SH. The strategy for SaN is divided into two phases that are synchronous to the two phases in PhN. Please see the illustration in Fig.3. The detailed multicast routing scheme is presented in Algorithm 1. To clarify the description, we first recall the formulation of the highway system.

$s_h(x, y)$: The yth horizontal slice in the xth horizontal slab in the scheme lattice $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$.

$s'_h(z)$: The zth row in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$.

$\hbar_h(x, y)$: The horizontal FH bearing the relay load initiated from the nodes in the slice $s_h(x, y)$.

$\hbar'_h(z)$: The horizontal SH bearing the relay load initiated from the nodes in the row $s'_h(z)$.

The formulations for the vertical case are similarly defined.

4) Analysis of Multicast Throughput for SaN: First, we should guarantee the priority of PhN in terms of throughput. Then, we propose the following theorem.

Theorem 5: Using Algorithm 1 to construct the multicast routing for SaN, denoted by $\mho_s^r$, and using the transmission scheme described in Fig. 3, denoted by $\mho_s^t$, to schedule SaN, we can protect the capacity of highways in PhN, including FHs and SHs, from decreasing in order sense due to SaN.

Proof: Due to the limited space and the similarity to the proofs of Lemmas 18 and 20 in [18], we omit the proof. Please refer it to Appendix B-C (Proof of Theorem 5) in [26]. ■

Because SaN does not add the load of any highway in PhN, by Theorem 5, we obtain that the presence of SaN has no impact on the order of throughput for PhN, when the strategy for SaN is designed as in Theorem 5.

Having ensured the priority of PhN, we now study the throughput for SaN. First, we answer Question 3 above.

Theorem 6: Under the multicast routing $\mho_s^r$ and transmission scheme $\mho_s^t$, all multicast sessions in SaN can be served.

Proof: In the second phase, the area of the *A-Type* PA is of order $o(\log n)$, while the area of the cell in the scheme lattice of the second phase in PhN, is of order $\Theta(\log n)$. Hence, for any secondary user (SU) in SaN, there exists a time slot $\tau(\tau_2, 2)$ during which this SU is not covered by any *active* PA. That is, $\bigcap_{\tau(\tau_2,2)} \mathcal{P}(\tau(\tau_2, 2)) = \emptyset$, where $1 \leq \tau_2 \leq K_2^2$, which completes the proof. ■

Now, we start to analyze the multicast throughput for SaN under $\mho_s^r$ and $\mho_s^t$ We first study the capacity of the FHs and SHs of SaN.

Theorem 7: Under the transmission scheduling $\mho_s^t$, the capacity of FHs and SHs in SaN can be achieved of order $\Omega(1)$.

Proof: The first phase in SaN has a TDMA scheduling period of $3K_1^2$. There are at least K_1^2 slots in which the sum interference produced by PhN at any receiver in SaN, denoted by I_{ps} is of a constant order, while the interference derived by SaN at this receiver, denoted by I_{ss}, is also summed up to a constant order. This means that PhN will not undermine the capacity of the FHs in SaN in order sense. Then, similar to the result of single dense networks in [24], the capacity of FHs in SaN can be achieved of order $\Omega(1)$.

The second phase in SaN has a TDMA scheduling period of $K_2^4 = K_2^2 \times K_2^2$. There are at least K_2^2 slots in which the corresponding cells in $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$ are really scheduled. During such slots, we can get that $I_{ps} = O(1)$ and $I_{ss} = \Theta(1)$, which means PhN will not undermine the

Algorithm 1 Multicast Routing based on FHs and SHs

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: **for** each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ **do**
2: According to the positions of u_i and u_j, determine the indexes a_i, b_i, y_i and c_j, d_j, x_j, where $u_i \in s_h(a_i, b_i) \cap s'_v(y_i)$; $u_j \in s_v(c_j, d_j) \cap s'_h(x_j)$.
3: Packets are drained from u_i into the horizontal FH $\hbar_h(a_i, b_i)$ via the vertical SH $\hbar'_v(y_i)$.
4: Packets are carried along the horizontal FH $\hbar_h(a_i, b_i)$.
5: Packets are carried along the vertical FH $\hbar_v(c_j, d_j)$.
6: Packets are delivered from the vertical FH $\hbar_v(c_j, d_j)$ to u_j along the horizontal SH $\hbar'_h(x_j)$.
7: **end for**
8: Considering the resulted routing graph, we merge the same edges (hops), remove those circles which have no impact on the connectivity of the communications for $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$. Finally, we obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

Algorithm 2 Multicast Routing based on Only SHs

Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: **for** each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ **do**
2: According to the positions of u_i and u_j, determine the indexes x_i and y_j, where $u_i \in s'_h(x_i)$; $u_j \in s'_v(y_j)$.
3: Packets are drained from u_i into the horizontal SH $\hbar'_h(x_i)$ by a single hop.
4: Packets are carried along the horizontal SH $\hbar'_h(x_i)$.
5: Packets are carried along the vertical SH $\hbar'_v(y_j)$.
6: Packets are delivered from the vertical SH $\hbar'_v(y_j)$ to u_j by a single hop.
7: **end for**
8: Using the similar procedure in Step 8 of Algorithm 1, we can obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

capacity of the SHs in SaN in order sense. Then, due to the *dense scaling*, we can derive the capacity of the SHs in SaN is of order $\Omega(1)$, [24].

Please see the detailed proof in Appendix B-D of [26]. ■

According to Theorem 7, we get Theorem 8.

Theorem 8: When $m_d = O(\frac{m}{(\log m)^2})$, the per-session multicast throughputs for SaN in the first phase and second phase, can be achieved of $\Omega(\frac{1}{\sqrt{m m_d}})$ and $\Omega(\frac{1}{m_d} \cdot (\log m)^{-\frac{3}{2}})$, respectively.

Proof: Please refer to Appendix B-E of [26]. ■

Like in the single *random dense network* [6], when the number of destination nodes of each multicast session is beyond some threshold, to be specific, $m_d = \Omega(\frac{m}{(\log m)^2})$, the multicast throughput derived by the multicast strategy based on the FHs and SHs cooperatively is not optimal in order sense. For this case, the multicast strategy based only on SHs can derive larger throughput. Next, we describe such routing scheme in Algorithm 2.

Theorem 9: By using the multicast routing based only on SHs, *i.e.*, the multicast routing constructed by Algorithm 2, and scheduling only for SHs, the per-session multicast throughput for SaN can be achieved of order

$$\begin{cases} \Omega(\frac{1}{\sqrt{m_d m \log m}}) & \text{when } m_d = O(\frac{m}{\log m}) \\ \Omega(1/m) & \text{when } m_d = \Omega(\frac{m}{\log m}) \end{cases}$$

Proof: According to Theorem 7, the capacity for each SH can be achieved of constant order. By a similar procedure of proof for Theorem 8, we can prove that the relay burden of each node in SHs is of order $O(\min\{m, \sqrt{m_d \cdot m \cdot \log m}\})$. Then, the theorem is proved. ■

Combining Theorem 8 and Theorem 9, we can get Theorem 2 by performing some simple algebraic computations.

C. When PhN Adopts BS-based Strategy

For this case, PhN adopts the classical BS-based strategy based on the scheme lattice $\mathbb{L}(\sqrt{n}, \sqrt{\frac{n}{b}}, 0)$, where $b = b(n)$ is the number of base stations in PhN. Under this strategy, the sources deliver the data to BSs during the Uplinks phase and BSs deliver the received data to destinations during the Downlinks phase. The communication between any pairs of PUs will be relayed by the BSs. In order to achieve better throughput, the BS-based strategy is adopted in PhN only when $b = \Omega(n/\log n)$ [5]. Because BSs are regularly placed, *i.e.*, each BS locates at the center of each cell, all cells can be simultaneously scheduled during both Uplinks phase and Downlinks phase, and sustain a rate of $(\frac{b}{n})^{\frac{\alpha}{2}}$ [5]. In each cell, all downlinks and uplinks are scheduled in sequence. All *B-Type* PAs will be always *active*. The SUs contained in such PAs cannot be served. Thus, we study the multicast capacity for SaN under the general definition of multicast capacity, *i.e.*, *asymptotic multicast capacity* (Definition 1). Obviously, the classic definition of capacity [13], [14], can be regarded as a special case of the *asymptotic multicast capacity*.

Next, we focus on the Downlinks phase in which SaN is scheduled. Whether or not SaN is scheduled in Uplinks phase has no impact on the multicast throughput in order sense.

1) Highway System: SaN still prefers to adopt the multicast strategy based on the FHs and SHs as in the case that all BSs are always *inactive*. Also, the *B-Type* PAs for BSs in the first phase and that in second phase are clusters of nine cells in the scheme lattices $\mathbb{L}(\sqrt{n}, c\sqrt{\frac{n}{m}}, \frac{\pi}{4})$ and $\mathbb{L}(\sqrt{n}, \sigma\sqrt{\log m \cdot \frac{n}{m}} - \epsilon_m, 0)$, respectively. Notice that the effectiveness of such *B-Type* PA relies closely on the fact that SaN is *dense scaling* while PhN is *extended scaling*.

Intuitively, as the number of BSs is increasing, such FHs or SHs as in the case absent of *active* BSs might be damaged by the PAs. Fortunately, the facts that $b = O(n)$ (then $b = o(\frac{m}{\log m})$) and BSs are regularly placed [5] guarantee SaN the same density of FHs, in order sense, as in the case absent of BSs. The original SHs in SaN are still remained. When the packets carried along an SH are stopped by a PA, they will detour the PA along the adjacent SHs. Hence, the loads of the SHs around the PAs are probably heavier than that of other

SHs. Please see the illustration in Fig. 4(b). Throughout the routing, the bottleneck in the second phase should be in those SHs with heavy burden. We will exploit this fact to analyze the multicast throughput for SaN.

2) Served Set of SaN: Now, we deal with the question how many SUs are not served at all. Denote the set of all SUs that are not served as $\mathcal{P}$, and denote the set of all sources in SaN by $\mathcal{S}$. Based on the sets $\mathcal{P}$ and $\mathcal{S}$, we propose a definition of the *served set* of multicast sessions, which can be divided into two regimes depending on m_d, *i.e.*, the number of destination nodes of each multicast session.

Definition 3 (Served Set): A *served set*, denoted by $\tilde{\mathcal{S}}$, is a subset of $\mathcal{S}$. Define $\tilde{\mathcal{S}} := \mathcal{S} - \mathcal{S} \cap \mathcal{P}$, when $m_d = \omega(\log m)$; and define $\tilde{\mathcal{S}} := \{v_{\mathcal{S},i} | \mathcal{U}_{\mathcal{S},i} \cap \mathcal{P} = \emptyset\}$, when $m_d = O(\log m)$.

We further define a subset of $\mathcal{D}_{\mathcal{S},i}$ as $\mathcal{D}'_{\mathcal{S},i} := \mathcal{D}_{\mathcal{S},i} - \mathcal{D}_{\mathcal{S},i} \cap \mathcal{P}$. Then, we have the following result.

Theorem 10: As $n \to \infty$, it holds that $|\tilde{\mathcal{S}}| \to |\mathcal{S}|$; and for each $v_{\mathcal{S},i} \in \tilde{\mathcal{S}}$, it holds that uniform *w.h.p.*, $|\mathcal{D}'_{\mathcal{S},i}| \to |\mathcal{D}_{\mathcal{S},i}|$.

Proof: Based on the fact that $n = o(\frac{m}{\log m})$, and by the tails of Chernoff bound [12], we can prove this theorem. Please refer to Appendix B-F of [26]. ■

For a multicast session $\mathcal{M}_{\mathcal{S},i}$, we define a subset of $\mathcal{U}_{\mathcal{S},i}$ as $\tilde{\mathcal{U}}_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \cup \mathcal{D}'_{\mathcal{S},i}$, and build the spanning tree $\mathrm{EST}(\tilde{\mathcal{U}}_{\mathcal{S},i})$ to guide the multicast routing of $\mathcal{M}_{\mathcal{S},i}$. According to Theorem 10, the throughput derived by the strategy based on the *served set* is *asymp-achievable*.

3) Guarantee of Priority of PhN: In the first phase of SaN, the interference produced by SaN at a receiving PU in PhN, denoted by I_{sp}, is of order $O(1)$, due to the setting of PAs. Then, $I_{sp} = O(N_0)$, where the constant $N_0 > 0$ is the ambient noise. Thus, the presence of SaN does not change the order of capacity of FHs in PhN. Similarly, we can prove that SaN does not impair the capacity of SHs in PhN in order sense.

4) Asymp-Achievable Multicast Throughput for SaN: We first consider the capacities of FHs and SHs in SaN.

Lemma 5: In the first phase, the sum of the interference produced by PhN at a receiver in SaN is of order $O(1)$.

Proof: The procedure of the proof is similar to that of Theorem 7. Please refer to Appendix B-G of [26]. ■

According to Theorem 7, during any time slot τ_1 in the first phase when a link $v_i \to v_j$ is scheduled, the interference on v_j produced by SaN itself can be bounded as $I_{ss}(v_i, v_j; \tau_1) = O(1)$. Combining with Lemma 5, we get that the capacity of FHs in SaN does not decrease due to PhN, and it is still of order $\Omega(1)$. Using the similar procedure, we can also prove that the capacity of SHs in SaN is still of order $\Omega(1)$. Next, we should analyze the loads of FHs and SHs, respectively. The former are obviously the same as the loads of FHs in SaN when BSs are absent. For the latter, the detouring scheme increases the loads of the SHs adjacent to PAs; but it can be proved that the increment does not change the order of the loads of those SHs non-adjacent to the PAs. Based on the analysis above, we can obtain Theorem 3, as one of our main results. The detailed proof of Theorem 3 can be found in Appendix B-H of [26].

D. When PhN Adopts Hybrid Strategy

In this case, the strategy for PhN can be divided into four phases: FHs phase, SHs phase, Downlinks phase and Uplinks phase [4], [5], [19]. We can use a simple *intermission-method* to achieve the same order of multicast throughput for SaN as in the case where no BS is adopted. Specifically, let SaN be idle during Downlinks phase and Uplinks phase of PhN, and schedule SaN in FHs phase and SHs phase of PhN by the similar schemes used in the case when PhN adopts the pure ad hoc strategy. Hence, the results of Theorem 2 and Theorem 4 for the case when PhN adopts hybrid strategy can be proved.

VI. Literature Review

The issue of capacity scaling laws for cognitive networks is a relatively new topic. In [3], the primary source-destination and cognitive S-D pairs were modeled as an interference channel with asymmetric side information. In [28], the communication opportunities were modeled as a two-switch channel. Note that both works [3], [28] had only considered the *single-user* case in which a single primary and a single cognitive S-D pairs share the spectrum. Recently, a *single-hop* cognitive network was considered in [29], where multiple secondary S-D pairs transmit in the presence of a single primary S-D pair. They showed that a linear scaling law of the *single-hop* secondary network is obtained when its operation is constrained to guarantee a particular outage constraint for the primary S-D pair.

For multi-hop and multiple users case, Jeon *et al.* [17] first studied the achievable unicast throughput for cognitive networks. In their cognitive model, the primary network is a random dense SANET or a dense BS-based network [5], and the secondary network is always a random dense SANET; two networks operate on the same space and spectrum. Following the model of [17], Wang *et al.* [30] studied the multicast throughput for the primary and secondary networks. In order to ensure the priority of primary users in meanings of throughput, they defined a new metric called *throughput decrement ratio* (TDR) to measure the ratio of the throughput of PaN in presence of SaN to that of PaN in absence of SaN. Endowing PaN with the right to determine the threshold of the TDR, they [30] devised the multicast strategies for SaN. Both the unicast routing in [17] and multicast routing in [30] are built based on the backbones similar to the *second-class highways* in [24], which suggests that the derived throughputs are not optimal under the Gaussian Channel model for most cases. By introducing percolation-based routing [12], [24], Wang *et al.* [18] improved the multicast throughput for the same cognitive network model as in [17], [30]; they showed that under some conditions, there exist the corresponding strategies to ensure both networks to achieve asymptotically the upper bounds of the capacity as they are stand-alone. One of the common characteristics in [17], [18], [30] is that the primary and secondary networks in all three models are dense scaling. More importantly, the common defect of three work is that all the strategies in [17], [18], [30] shield the time-domain, which makes the routing path always detour the protection areas (or preservation regions), although they are sometimes *inactive*. Under those strategies, there possibly some secondary users that can never be served.

VII. Conclusion

We study the multicast capacity for cognitive networks that operate under TDMA scheme. The network model consists of the primary hybrid network (PhN) and the secondary ad hoc network (SaN). PhN and SaN are assumed to be *extended scaling* and *dense scaling*, respectively, which enhances the reality of our network model. We devise the dynamic *protection area* (PA) for each primary node according to the strategy adopted in PhN. Based on the PAs, we design the multicast strategies for SaN under which the highway system acts as the multicast backbone. Under the precondition that SaN should have no negative impact on the order of the throughput for PhN, our strategy has the following merits. Firstly, by our strategy, the optimal throughput for SaN can be (asymptotically) achieved for some cases. Secondly, under our strategy, unlike most related works, secondary nodes can access opportunistically into the spectrum from both time and space domains. Thirdly, under our strategy, all secondary users can be served, except for the case that PhN adopts BS-based strategy.

Acknowledgments

The research of authors is partially supported by the National Basic Research Program of China (973 Program) under grants No. 2010CB328100 and No. 2010CB334707, the Program for Changjiang Scholars and Innovative Research Team in University, the Shanghai Key Basic Research Project under grant No. 10DJ1400300, the Expo Science and Technology Specific Projects of China under grant No. 2009BAK43B37, the NSF CNS-0832120, the National Natural Science Foundation of China under grant No. 60828003, the Program for Zhejiang Provincial Key Innovative Research Team, and the Program for Zhejiang Provincial Overseas High-Level Talents.

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