1. Let $A = \{a, \{\varnothing\}\}$. Mark with true (T) or false (F) each of the following statements:

a)	$a \in A$	**T**	b)	$\{a\} \in A$	**F**
c)	$a \subseteq A$	**F**	d)	$\{a\} \subseteq A$	**T**
e)	$\varnothing \subseteq A$	**T**	f)	$\varnothing \in A$	**F**
g	$\{\varnothing\} \subseteq A$	**F**	h)	$\{\varnothing\} \in A$	**T**
i)	$\{\varnothing\} \in power(A)$	**F**	j)	$\{A\} \in power(A)$	**F**

2. Let $S = \{0, 1, 2, 3, 4\}$ and a relation R on S defined as

$a \; R \; b$ if and only if $b = a^2$, $a, b \in S$

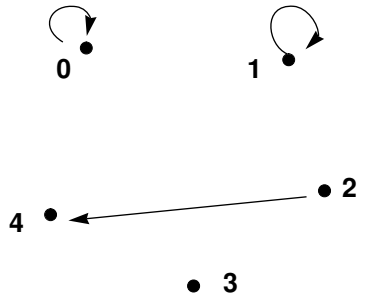
a) show the set representation of R

R = {<0,0>, <1,1>, <2,4>}

b) show the matrix representation of R Show the digraph of R

1	**0**	**0**	**0**	**0**		
0	**1**	**0**	**0**	**0**		
0	**0**	**0**	**0**	**1**		
0	**0**	**0**	**0**	**0**		
0	**0**	**0**	**0**	**0**		

The matrix of R. Clearly mark the limits of the matrix



The digraph of R

c) mark with true (T) or false (F) the following statements

Statement	Your answer	Comments
R is reflexive	**F**	**Some 0's on main diagonal**
R is not reflexive	**T**	
R is irreflexive	**F**	**Some 1's on main diagonal**
R is symmetric	**F**	**The matrix is not symmetric**
R is asymmetric	**F**	**Some 1's on main diagonal**
R is antisymmetric	**T**	**Inspect the matrix**
R is transitive	**F**	**Inspect R**
The digraph of R is acyclic	**F**	**Cycle of length 1 in 0**

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Statement	Your answer	Comments
There is a path of length 3 in the digraph of R	**T**	**π: 0000**

3. Let A = {all people living in the U.S.A who receive Medicare[1]} and B = {x | x is a nine digit number}. The function f defined on A with values in B, associates each person in A her/his Social Security Number.
a) decide whether the function is injective or not. Explain.

Yes: different people have different Social Security Numbers (by the definition of SSN)

b) decide whether the function f is surjective or not. Explain.

No: there are Social Security Numbers that correspond to people who do not receive Medicare.

4. A parking lot has 11 visitor spaces, numbered from 0 to 10. Visitors are assigned parking spaces using the hashing function $h(n) = n$ **mod** 11 where n is the number formed from the last three digits on the visitor's license plate.
a) Several cars arrive in the order indicated below. What spaces are they assigned? In case of conflicts use linear probing with a gap of 2.

Car number	Assigned space
111	**1**
274	**10**
403	**7**
133	**3**
286	**0**

5. A recursive procedure that does the *inorder* listing of a binary tree is presented below:

```
// Inorder listing of a binary tree: list left subtree, then
// list root, then list the right subtree.
//
void InorderList(Tree t)
{
Node r;
      r = root(t);
      if (r == LEAF) {
            print(r);
            return;
      }
```

1. Medicare is a U.S. government program of medical insurance for aged or disabled persons

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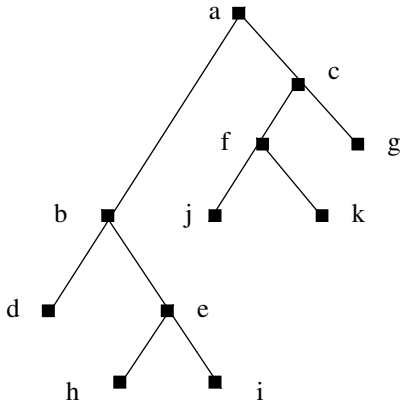
```
        InorderList(left(t));
        print(r)
        InorderList(right(t));
}
```

What would the procedure print if called with a tree `t` like the one in the figure below?



dbheiajfkcg

6. For each of the grammars listed in the following table, indicate whether it is context free (mark with a Y) or not (mark with a N)

Grammar	Your answer
$S \rightarrow aAB,\ A \rightarrow Bb,\ B \rightarrow \lambda$	**Y**
$S \rightarrow ABa,\ AB \rightarrow b$	**N**
$S \rightarrow aA,\ aA \rightarrow B,\ B \rightarrow aA,\ A \rightarrow b$	**N**
$S \rightarrow AB,\ B \rightarrow bAb,\ bAb \rightarrow b$	**N**
$S \rightarrow \lambda$	**Y**

7. G is a context free grammar defined by:

$N = \{S, A\}$ $T = \{0, 1\}$ Start symbol: S

Productions:

$S \rightarrow 0S \mid 1A \mid 1 \mid \lambda$

$A \rightarrow 1A \mid 1$

b) what is the language of the grammar?

$L(G) = \{ 0^n1^m \mid n \geq 0, m \quad 0 \}$

a) find a leftmost derivation of the string 00011

$S \Rightarrow 0S \Rightarrow 00S \Rightarrow 000S \Rightarrow 0001A \Rightarrow 00011$

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b) is the grammar G ambiguous? Explain.

No: it is not possible to use different rules to obtain the same sentential form in a derivation.

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