December 12, 2002

v1

cs330 - Discrete Structures
Fall 2002

Final Exam
closed books, closed notes

Starts: **8:00 am**   Ends**: 10:00 am**

Name:_____(please print)

ID:_____

Problem	Max points	Your mark	Comments
1	5		
2	10		
3	5		
4	5		
5	10		10*1
6	15		5+5+5
7	5		
8	10		
9	5		
10	10		
11	40		8*5
	120		

This paper will be retained for one year in the Computer Science Department.

Do NOT write on the back of any page!

1. A *boolean function* is a function whose domain and codomain is the set $\{0, 1\}$. List all boolean functions of one variable.

2. Assume the function f of type $Lists(S) \rightarrow Power(S)$ defined by

```
f(<x₁, x₂, ..., xₙ>) = {x₁, x₂, ..., xₙ}
```

where *Lists(S)* is the set of all lists over the finite set *S*. Is f bijective? Explain.

3. Consider the set *S* of all functions of type $N \to \{0, 1\}$, where *N* is the set of natural numbers. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 1/3 of the credit for this problem).

4. Find a regular expression for the language consisting of strings that begin and end with the same letter over the alphabet {a, b}.

5. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	1010 belongs to the language (T/F)	10001 belongs to the language (T/F)
$1^*0^*1^*$		
$(1+0)^*(1)^*$		
$(0+1+\varepsilon)^*1+(01)^*1$		
$(00)^*1^*(10)1$		
$(0+1)+(10+1)^*$		

6.   This is the postfix (reverse Polish) notation for an algebraic expression:

```
xy+zw*uy/--x*
```
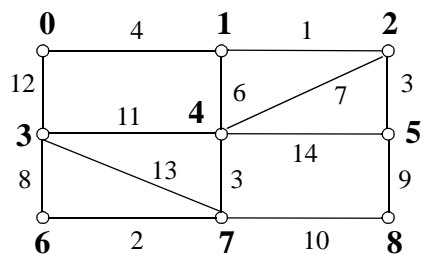
a) Show the tree representation of this expression.

b) Show the corresponding algebraic expression

c) Show the prefix notation for the expression.

7. Construct the minimum spanning tree for the graph below:

Minimum weight =

8. Let *F(x, y)* be the statement "*x* can fool *y*" where the universe of discourse is the set of all people in the world. Use quantifiers to formalize each of the following statements:

a) "Everybody can fool John"

b) "Somebody can fool everyone"

c) "Every politician can fool John" (assume that *P(z)* is the statement "*z* is a politician")

9.   Which of the following functions grows faster? Explain.

- $f_1(n) = 0.01^n - n^{1000000}$
- $f_2(n) = n^{100} + n^{50} + 1000000$

10.   a) Decide whether the relation *startsWithSameDigit* is an equivalence relation on the set S of all natural numbers smaller than or equal to 20.

b) If the relation is an equivalence relation, then what is the partition it establishes on S ?

11. Give a definition for:

a) Implication

b) Relation on a set

c) Time complexity of an algorithm

d) Tree

e) Alphabet

f) Language

g) Regular Language (the inductive definition)

Function